Correspondence Arguments between Resonance and Parton Pictures in Deep Inelastic Processes and the Patterns of Scaling Violation

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The patterns of scaling violation of the deep inelastic structure function are investigated on the basis of the resonance picture. The correspondence arguments between two complementary pictures, i. e., the resonance and the parton pictures, have enabled us to discriminate various field theories. Our main results are that i) the conventional field theory is incompatible with the resonance picture, thus ruled out as a candidate for the strong interactions, ii) QCD and the new scaling law may have the capability to unify the above two pictures. Discussions about the correlation between the resonance and the valence- (sea-) quark contribution are given. We also show that the solution satisfying the correspondence requirement give excellent fits to the data for the proton magnetic form factor.

I. Introduction

In the early stage of the attempts to understand the phenomenon associated with Bjorken scaling, the parton picture[1)] and the resonance picture[2),3),4)] had been used as two theoretical bases which might be expected to complement each other. The observation of "duality" in electron-nucleon scattering by Bloom and Gilman[2)] had clearly indicated the importance of this complementarity.

In recent years significant deviation from Bjorken scaling has been observed both in the experiments of electron[5)] and muon scattering[6)] off nucleon, and also in neutrino reactions.[7)] Many of the recent attempts to understand the scaling violation have been mainly based on the quark-parton model,[1),8)] or on the renormalizable field theories[9)-13)] whereas there is hardly any attempt based on the resonance picture. In this sense, among the two complementary pictures only the parton picture has made a great progress; it has led to the quark-parton model and to the understanding of scaling violation in terms of various field theoretical approaches, especially quantum chromodynamics (QCD). This picture has also added some understanding to the resonance-saturation picture (or the Bloom-Gilman duality) in QCD.[10] In spite of the success of QCD, however, up to now experimental results cannot conclusively decide which type of field theory is to be favoured.[14] Moreover, recently an interesting scaling law was proposed,[15] which was found to give a beautiful description of the observed pattern of scaling violation.[16] In view of the above situation, it seems worthwhile to pursure more deligently the aforementioned complementarity between the resonance and the parton pictures.

In order to make this complementarity more powerful and predictive, guided by the Bloom-Gilman duality, let us hypothesize the following *correspondence*: The description of the structure function based on the resonance picture should coincide with the one based on the parton picture in the limit of large momentum transfer, and the approach to this asymptotic description should occur in a way that in the finite (small) momentum transfer region the structre function based on the parton picture at least well average the one based on the resonance picture. Investigations of the scaling violation based on the resonance picture[17] and the description of the proton magnetic form factor in terms of the solutions satisfying the correspondence requirement[18] have been done and the usefulness of the correspondence arguments was demonstrated.

In this paper we review how to formulate the correspondence requirement mathematically, and how it works to discriminate various field theories (sections II and III). These studies are performed by taking the moment of the structre function. We give some discussions on the structure function itself in the resonance picture (section IV). We also show that the resonance form factors which satisfy the correspondence requirement can give excellent descriptions of the experimentally observed behaviour of the proton magnetic form factor in the large momentum transfer regions (section V).

II. Moment Sum Rules in the Small-Q^2 Region and the Resonance Saturation

A. Kinematics and Definitions.

In order to study the consequences of the correspondence between the parton or the field theoretical and the resonance pictures, let us briefly present the theoretical framework of the deep inelastic reactions based on field theories.

Inelastic lepton-nucleon scattering is described by the absorptive part of the forward current-nucleon amplitude

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{2\pi}\int d^4x e^{iq\cdot x}\langle p|[J_\mu(x), J_\nu(0)]|p\rangle \\ &= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)W_1(\nu, q^2)+\frac{1}{m^2}\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)W_2(\nu, q^2) \\ &= \frac{1}{\pi}\text{Abs. } T_{\mu\nu} \end{aligned} \tag{1}$$

$$T_{\mu\nu} = i\int d^4x e^{iq\cdot x}\langle p|T(J_\mu(x)J_\nu(0))|p\rangle. \tag{2}$$

In the language of field theory, deep inelastic experiment measure the light-cone singularities of the current commutator (or product) which appears in Eqs. (1) and (2).

This product of two current operators can be analyzed in terms of the operator product expansion,[19] which reads

$$\int d^4x e^{iq\cdot x}\langle p|T(J_\mu(x)J_\nu(0))|p\rangle$$
$$=t_{1\mu\nu}\sum_{n=0}^{\infty}\langle p|O_{\mu_1\cdots\mu_n}(0)|p\rangle\int d^4x e^{iq\cdot x}\Delta_{1n}(-x^2+i\varepsilon)x^{\mu_1}\cdots x^{\mu_n}$$
$$+t_{2\mu\nu}\sum_{n=0}^{\infty}\langle p|O^{\alpha\beta}_{\mu_1\cdots\mu_n}(0)|p\rangle\int d^4x e^{iq\cdot x}\Delta_{2n}(-x^2+i\varepsilon)x^{\mu_1}\cdots x^{\mu_n} \tag{3}$$

where

$$t_{1\mu\nu}=q_\mu q_\nu-q^2g_{\mu\nu}$$
$$t_{2\mu\nu}=g_{\mu\alpha}q_\beta q_\nu+g_{\beta\nu}q_\mu q_\alpha-g_{\mu\alpha}g_{\nu\beta}q^2-g_{\mu\nu}q_\alpha q_\beta. \tag{4}$$

The moment sum rules which may hold in the limit of large Q^2 $(=-q^2)$ are

$$M_1(n,Q^2)=\int_1^\infty d\omega\,\omega^{-n-3}F_1(\omega,Q^2)=-A_{1,n+1}\cdot E_{1,n+2}(Q^2)+A_{2,n}\cdot E_{2,n}(Q^2),$$
$$M_2(n,Q^2)=\int_1^\infty d\omega\,\omega^{-n-2}F_2(\omega,Q^2)=A_{2,n}\cdot E_{2,n}(Q^2), \tag{5}$$

where $\omega=2m\nu/Q^2$ and $F_i(\omega,Q^2)$ $(i=1,2)$ are the usual structure functions defined by

$$F_1(\omega,Q^2)=W_1(\nu,Q^2),\quad F_2(\omega,Q^2)=\nu W_2(\nu,Q^2)/2m,$$

where m is the mass of target nucleon. $A_{i,n}$ $(i=1,2)$ are the reduced matrix elements of the local operators $O_{\mu_1\cdots\mu_n}$ and $O^{\alpha\beta}_{\mu_1\cdots\mu_n}$ between the spin-averaged nucleon states, e. g..

$$\langle p|O^{\alpha\beta}_{\mu_1\cdots\mu_n}(0)|p\rangle=i^n(p^\alpha p^\beta p_{\mu_1}\cdots p_{\mu_n}-\text{trace terms})\cdot A_{2,n}, \tag{6}$$

and $E_{i,n}(Q^2)$ are essentially the Fourier transform of the c-number singular functions $\Delta_{in}(-x^2+i\varepsilon)$ appearing in the operator product expansion, Eq. (3),

$$E_{i,n}(Q^2)=a_i\cdot(Q^2)^{n+i}\left(\frac{\partial}{\partial Q^2}\right)^n\int d^4x e^{iq\cdot x}\Delta_{in}(-x^2+i\varepsilon), \tag{7}$$

where $a_1=1/2$, $a_2=1/8$. Various theories, such as the conventional field theories with ultra-violet fixed point (CFT),[13] QCD[9),10] and the new scaling law (NSL),[15),16] give particular prediction about the Q^2-dependence of $E_{i,n}(Q^2)$, i. e., of the moment integral of the relevant structure functions. The patterns of scaling violation predicted by the above three theories are*)

$$M_2(n,Q^2)=M_2(n,Q_0^2)\cdot\left(\frac{Q^2}{Q_0^2}\right)^{-\gamma(n)}$$
$$\gamma(n)=\begin{cases}a\left[1-\dfrac{2}{(n+2)(n+3)}\right], & \text{CFT} \quad (8a)\\ b(n+1), & \text{NSL} \quad (8b)\end{cases}$$

and

$$M_2(n,Q^2)=M_2(n,Q_0^2)\cdot\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right)^{-\gamma(n)}$$
$$\gamma(n)=G\left[1-\frac{2}{(n+2)(n+3)}+4\sum_{j=0}^{n}\frac{1}{j+2}\right],\quad \text{QCD}. \tag{8c}$$

*) For simplicity hereafter we only consider the moment integral of the structure function $F_2(\omega,Q^2)$, i. e., $M_2(n,Q^2)$.

Therefore we can test the theories by studying the Q^2-dependence of the moment, which is essentially that of $E_2(Q^2)$.

B. Moment Sum Rules in the Small Q^2 Region.

In order to apply the resonance-saturation program to the framework reviewed in the previous section, we here extrapolate the moment sum rules, Eqs. (5), to the small Q^2 region.

We start with the following experimental observations. When Q^2 is small (say, $Q^2 \lesssim 1$ GeV^2) $F_2(\omega, Q^2)$ shows resonance structure whereas it becomes a smooth function of ω when Q^2 becomes large (say $Q^2 \gtrsim 4$ GeV^2). We call, for the sake of brevity, this smooth function of ω the "asymptotic structure function", $F_2^{AS}(\omega, Q^2)$. The Q^2-dependence of $F_2^{AS}(\omega, Q^2)$ is, by definition, completely specified by the moment sum rules. Then the generalized version of the Bloom-Gilman duality says the following: When we extrpolate the asymptotic structure function down to the small Q^2 region (of order of 1 GeV^2) through Eq. (5), replacing the original Bjorken's variable ω by the Bloom-Gilman's $\omega'=\omega+m^2/Q^2$,*) then the extrapolated structure function well averages the observed structure function, in a semi-local sense. With the aid of this, we can obtain the moment sum rules at small Q^2 region,

$$\begin{aligned} M_2(n, Q^2) &= A_{2,n}\cdot E_{2,n}(Q^2) \\ &= \int_1^\infty d\omega'(\omega')^{-n-2}F_2^{AS}(\omega', Q^2) \\ &= \left(\frac{2m}{Q^2}\right)^{-n-2}\int_{\nu_0(Q^2)}^\infty d\nu\left(\nu+\frac{m}{2}\right)^{-n-2}\frac{\nu}{2m}W_2(\nu, Q^2), \quad n\geqq 0, \end{aligned} \tag{9}$$

where $\nu_0(Q^2)=Q^2/2m$.

C. Resonance Saturation

In the resonance model we have (in the narrow-width approximation)

$$W_2(\nu, Q^2)=(2m)^2\sum_k w_k(Q^2)\,\delta(2m\nu+m^2-Q^2-M_k^2), \tag{10}$$

$$w_k(Q^2)=g_k^2[G_k(Q^2)]^2. \tag{11}$$

In Eq. (11), $G_k(Q^2)$, the form factor of the resonance with mass M_k, is assumed to be the same for all resonances with the same masses, and g_k^2 is the effective coupling constant which is the coupling constant multiplied by the resonance density, i. e.,

$$g_k^2=\sum_\eta \rho_\eta(M_k^2)\,\kappa_\eta^2. \tag{12}$$

The sum is taken over different quantum numbers η, specifying each resonance with the same mass M_k, e.g., spin J and radial quantum number N etc. The moment sum rules then give

$$M_2(n, Q^2)=\sum_k g_k^2[G_k(Q^2)]^2\left[1+\frac{M_k^2}{Q^2}\right]^{-n-2}\left[1+\frac{M_k^2-m^2}{Q^2}\right]. \tag{13}$$

*) The ξ-variable might be more appropriate than ω'. For present purposes, however, there is no difference between the two variables and we use ω' for simplicity. As for the ξ-variable, see, O. Nachtmann, Nucl. Phys. **B 63** (1973) 237; H. Georgi and H. D. Politzer, Phys. Rev. Letters **36** (1976) 1281.

Here we need the knowledge about (i) the behaviour of the resonance form factor $G_k(Q^2)$, (ii) the effective coupling constants g_k^2 and (iii) the formula for the resonance masses. The experience with approximate scaling in the resonance models[3),4)] and the Bloom-Gilman duality almost uniquely require that the resonance form factor should obey the following "scaling formula" with the approximate dipole behaviour for large Q^2,

$$G_k(Q^2)=G\left(\frac{Q^2}{M_k^2}\right)\simeq\left(1+\frac{Q^2}{M_k^2}\right)^{-d/2},\quad d\cong 4, \tag{14}$$

and that the effective coupling constants should behave as $g_k^2 \sim 1/k$. The formula (14) has also some experimental supports,[20)] especially for the point that $G_k(Q^2)$ may become a function of the scaled variable $\lambda=Q^2/M_k^2$. Therefore at first we consider the form factor

$$\text{(A)}\quad G_k(Q^2)=\left[1+\frac{Q^2}{M_k^2}\right]^{-d(Q^2)/2}. \tag{15a}$$

The only ambiguity which would give rise to an important effect is an $1/Q^2$ correction to Eq. (15a). This correction may be reasonably taken into account with the form factor

$$\text{(B)}\quad G_k(G^2)=\left[1+\frac{M_k^2}{Q^2+m^2}\right]^{\delta/2}\left[1+\frac{Q^2}{M_k^2}\right]^{-d(Q^2)/2}. \tag{15b}$$

In Eq. (15a, b), $d(Q^2)$ is an arbitrary function of Q^2, and it will be discussed later. We also take[3),4)]

$$g_k^2/g_0^2=1/(k+a), \tag{16}$$

which is a simple representation of the behaviour $g_k^2 \sim 1/k$. As for the mass formula, we take the quadratic one*)

$$M_k^2=m^2+\mu^2 k,\quad k=1,2,\cdots. \tag{17}$$

Here we notice that in the earlier work on resonance models,[3),4)] $d(Q^2)$, the power of the resonance form factor, was assumed to be independent of Q^2 and was taken to be equal to 4. Such an assumption seems to be too restrictive. We therefore prefer to let $d(Q^2)$ have a general Q^2-dependence.

III. Large-Q^2 Behaviour of the Moment and the Patterns of Scaling Violation

In this section we study the large-Q^2 behaviour of the moment, Eq. (13), which was originally obtained by saturating the structure function with resonances in the small-Q^2 region. According to the correspondence requirement, the description of the moment in the resonance saturation picture should coincide with those predicted by field theories in the asymptotic limit of large-Q^2.

A. General Results.

*) The linear mass formula, $M_k=m+\mu' k$, is also a reasonable one. This, however, gives essentially the same result as the quadratic one, and we hereafter consider only Eq. (17).

Inserting Eqs. (15), (16) and (17) into the expression of the moment, for large-Q^2 we can approximate the sum by an integral with the variable $z=(\mu^2k+m^2)/Q^2$ to get

$$M^2(n, Q^2)=g_0^2\int_{m^2/Q^2}^{\infty}\frac{dz}{z}\Big(1+\frac{1}{z}\Big)^{-d(Q^2)}(1+z)^{-n-2}\Big(1+z-\frac{m^2}{Q^2}\Big), \tag{18a}$$

or

$$M^2(n, Q^2)=g_0^2\int_{m^2/Q^2}^{\infty}\frac{dz}{z}\Big(1+\frac{1}{z}\Big)^{-d(Q^2)}\Big[1+z\Big(1+\frac{m^2}{Q^2}\Big)\Big]^{\delta}(1+z)^{-n-2}\Big(1+z-\frac{m^2}{Q^2}\Big). \tag{18b}$$

In the limit of large Q^2 ($\gg m^2$), we may safely set $m^2/Q^2\cong 0$ in Eqs. (18a, b) and obtain

$$\begin{aligned} M_2(n, Q^2)&=g_0^2\int_0^1 dx\, x^n(1-x)^{d(Q^2)-1}\\ &=g_0^2\cdot B(n+1, d(Q^2)), \end{aligned} \tag{19a}$$

or

$$M_2(n, Q^2)=g_0^2\cdot B(n+1-\delta, d(Q^2)), \tag{19b}$$

where $x^{-1}=1+Q^2/M_k^2$ is essentially the Bloom-Gilman variable. Eqs. (19) say in general that if $d(Q^2)$ is independent of Q^2, as predicted by the CFT,[21] then we should have the Bjorken scaling in the limit of largeQ^2. On the other hand, since CFT actually also predicts scaling violation,[18] we must say that the resonance picture is incompatible with CFT. According to the correspondence statement, this result indicates that CFT cannot unify the resonance and the parton pictures and, in this spirit, it seems to be ruled out as a likely candidate for the underlying strong interactions.

B. Patterns of Scaling Violations.

Here we consider the interesting case where the power of the resonance form factor, $d(Q^2)$, depend explicitly on Q^2 and increases indefinitely as Q^2 goes to infinity. In fact, QCD predicts this behaviour with $d(Q^2)$ behaving roughly as[22]

$$d(Q^2)=4G\ln\Big[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\Big]+p+1, \quad G=4/27, \tag{20}$$

where p is related to the threshold behaviour ($\omega\sim 1$) of the structure function at $Q^2=Q_0^2$

$$F_2(\omega, Q_0^2)\underset{\omega\sim 1}{\sim}(\omega-1)^p. \tag{21}$$

Let us consider the two models separately.

B–1) Model A.

If $d(Q^2)$ increases indefinitely as Q^2 goes to infinity, then we have

$$M_2(n, Q^2)\sim[d(Q^2)]^{-n-1}\Gamma(n+1) \text{ as } Q^2\to\infty. \tag{22}$$

Insofar as the expression $d(Q^2)$ given by QCD, Eq. (20), can be accommodated in the resonance saturation picture and gives the scaling violation, QCD is compatible with the resonance picture. This fact partly confirms the explanation of the Bloom-Gilman duality by QCD.[10] As is easily seen, however, the behaviour of the moment

$$M_2(n, Q^2)\underset{Q^2\to\infty}{\sim}(\ln t)^{-n-1}, \quad t=\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \tag{23}$$

is not exactly the one predicted by QCD

$$M_2(n, Q^2) \sim t^{-a \cdot \ln(n+2)}. \tag{24}$$

Note, however, that if we take the limit $n \to \infty$ of the moment Eq. (19a), we have

$$M_2(n, Q^2) \underset{n\to\infty}{\sim} \Gamma(d(Q^2))\, n^{-d(Q^2)},$$

which gives (by using Eq. (20))

$$\begin{aligned} &M_2(n, Q^2) \underset{\substack{n\to\infty \\ Q^2\to\infty}}{\sim} f(t)\, t^{-4G \ln(n)}, \\ &f(t) = t^{-1} (\ln t)^{4G \ln t + p + 1/2}. \end{aligned} \tag{25}$$

This behaviour is quite similar to the one predicted by QCD, Eq. (24), which means the behaviour of the moment at least in the large n and large Q^2 region is quite consistent in both QCD and the resonance picture.

Another interesting case is provided with the choice of $d(Q^2)$

$$d(Q^2) \sim (Q^2/Q_0^2)^{\alpha}, \quad \alpha > 0. \tag{26}$$

Then the moment integral behaves as

$$M_2(n, Q^2) \sim (Q^2/Q_0^2)^{-\alpha(n+1)}, \tag{27}$$

which is *exactly the same* pattern of scaling violation predicted by the new scaling law (NSL)[15),16)] with the anomalous dimension $\gamma(n)$

$$\gamma(n) = \alpha(n+1). \tag{28}$$

This solution seems to be quite intersting on the basis of the correspondence principle and we should pay special attention to this new scaling law.

It is worthwhile to take notice of the following observation. Eq. (22) suggests that $M_2(n, Q^2)$ may become independent of Q^2 when $n = -1$ (not when $n = 0$). In the language of the quark-parton model, it is not the total number of quarks and antiquarks, but the number of valence-quarks which is conserved. Therefore, the Q^2-independence of the moment $M_2(n, Q^2)$ at $n = -1$ indicates that *in Model A* resonances can build up only the valence-quark component of the structure function.

B–2) Model B.

In this model, the moment is given by Eq. (19b) which behaves

$$M_2(n, Q^2) \sim [d(Q^2)]^{-n-1+\delta} \Gamma(n+1-\delta), \text{ as } Q^2 \to \infty, \tag{29}$$

when $d(Q^2)$ increases indefinitely as Q^2 goes to infinity. We have here, with Eq. (20),

$$M_2(n, Q^2) \sim (\ln t)^{-n-1+\delta}, \quad t = \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}. \tag{30}$$

If we take the large-n limit as well as the limit $Q^2 \to \infty$ then we obtain

$$M_2(n, Q^2) \underset{\substack{n\to\infty \\ Q^2\to\infty}}{\sim} f(t)\, t^{-4G \ln(n-\delta)}, \tag{31}$$

where f(t) is the same as the one in Eq. (25).

By using Eq. (26) we also have the pattern

$$M_2(n, Q^2) \sim (Q^2/Q_0^2)^{-\alpha(n+1-\delta)}. \tag{32}$$

Here we take notice of the fact that Eq. (29) suggests the possibility that the moment becomes independent of Q^2 when $n=\delta-1$. In this sense we pay a special interest to the case $\delta=1$. In this case (Model B with $\delta=1$) the behaviour of the moment (30), (31) may be consistent with the prediction of QCD with the singlet operator contributions. The pattern (32) is exactly the same as the prediction of the NSL with the anomalous dimension

$$\gamma(n)=\alpha n. \tag{33}$$

In the language of the field thcory, the constancy of the zeroth moment of the structure function means that the structure function is completely dominated by the energy-momentum tensor and its higher rank operators. According to the correspondence argument, therefore, *in Model B with* $\delta=1$ resonances may build up only the t-channel singlet or the sea-quark component of the structure function. This possibility seems somewhat different from the conventional understanding of the correlation between resonances and valence-quarks. From our examples, however, it becomes clear that the asymptotic dipole-like behaviour of the resonance form factor does not necessarily lead to the expected resonance-valence-quark relation.

Finally we give some arguments on the valence-quark contribution in this model. Consider the case where all resonances have the same dipole-like form factors (15b) with $\delta=1$. The valence-quark contribution is most easily extracted out by taking the difference of the structure function between proton and neutron targets. Let us assume, for simplicity, that there exist two towers of resonances with masses M_k^2 and $M_k'^2=M_k^2+\kappa^2$, whose sum and difference contribute to the structure functions of the sum and difference of proton and neutron, respectively. Then the sum will build up the sea-quark contribution, Eq. (29), with $\delta=1$, and the difference will give the moment

$$\begin{aligned} M_2^V(n, Q^2) &= -\frac{\kappa^2}{Q^2}\int_0^\infty dz\frac{d}{dz}[z^{d(Q^2)-1}(1+z)^{-d(Q^2)-n}] \\ &\cong \kappa^2 Q^{-2}[d(Q^2)]^{-n-1}\Gamma(n+2). \end{aligned} \tag{34}$$

This seems to be satisfactory as the valence-quark contribution. The more interesting result in the above discussion is the fact that by slightly improving the treatment of the resonances there appears in fact the factor $n+1$ which changes the original Γ-factor in Eq. (29) $\Gamma(n+1)$ to $\Gamma(n+2)$. Therefore the above result is quite encouraging, at least to expect some mechanisms which may change the original Γ-factors in Eqs. (22), (29).

C. Relation between the Resonance and the Valence- or Sea-Quark Contribution.

In the previous subsection B, we have briefly mentioned the possibility that in Model A (Model B) the moment $M_2(n, Q^2)$ becomes independent of Q^2 when $n=-1$ $(n=0)$ and therefore the resonances can build up only the valence- (sea-) quark contribution to the structure function. To be precise, however, in Model A the factor $\Gamma(n+1)$ appearing in Eq. (22) has a pole at $n=-1$ and this may invalidate the above-mentioned Q^2-independence of the moment at $n=-1$. The same problem occurs in Model B because of the factor $\Gamma(n)$ in Eq. (29) with $\delta=1$.

The above problem concerning the pole at $n=-1$ $(n=0)$ might probably the ficticios one and it could be due to our oversimplified treatment of the contributions from various resonances to the structure function. In fact, as was discussed in B-2, by considering the contribution from the difference of two series of resonances, the factor $n+1$ really emerges which correctly cancels the pole at $n=-1$. Therefore it seems to be reasonable to expect, when we know the details of the various resonance-contributions, the poles coming from the Γ-factors may disappear. We take this possibility and investigate the consequences.

Then in Model A resonances may build up only the valence-quark contribution. We here note that it is the *non-exotic resonance* whose form factor is known to behave roughly as *dipole*, Eq. (15a). So the above correspondence may be refined as the correlation between the non-exotic resonances and the valence-quarks. In Model B, resonances may build up the sea-quark contribution and also it may be possible to construct the valence-quark one. Note, however, in the preceeding discussions it is implicitely assumed that all resonances have asymptotically the same Q^2-behaviour, namely, $d(Q^2)$ is nearly equal to 4 at moderate values of Q^2 in Eqs. (15).

There is also another interesting possibility. As can be easily seen by Eqs. (19a, b), both models A and B satisfy the Drell-Yan-West relation.[23] This fact and the sharp decrease of the sea-quark distribution near $x=1$ compared to the valence-quark one suggest that there exist two types of resonances whose form factor behave differently as $Q^2\to\infty$. In fact, recently there have been several theoretical discussions and also some experiemental evidences of the *exotic resonances*.[24],[25] If these exotic resonances are in fact hadronic bound states and correspond to the configuration qqqq$\bar{\text{q}}$ or six quarks, then these form factors should behave roughly as $(Q^2)^4$ or $(Q^2)^5$ by the quark counting rule. This behaviour of the form factor gives roughly the consistent behaviour of the sea-quark distribution near $x=1$.[*] If we take the above seriously, then the following seems to be quite interesting. The ordinary non-exotic resonances have the form factor Eq. (15a) ($d(Q^2)\simeq 4$ at $Q^2\simeq 2-4$ GeV^2) and build up the valence-quark contribution, whereas the exotic resonances have the form factor Eq. (15b) ($d(Q^2)\simeq 8$ or 10 at $Q^2\simeq 2-4$ GeV^2) and they build up the sea-quark component.

IV. Structure Function at Large Q^2

We have studied the properties of the moment of the structure function in the previous section. Here we consider the asymptotic description of the structure function itself in the present resonance saturation picture. Our results presented in this section are all confined to the large Q^2 region ($Q^2\gg m^2$).

In Model A, we have obtained the moments, Eq. (19a),

*) The sea-quark distribution behaves near $x=1$ as

$$F_2^s(x, Q_0^2)\sim(1-x)^{p'},\quad p'\gtrsim 6 \text{ at } Q_0^2\cong 2-4 \text{ GeV}^2,$$

which means $d(Q^2)$ in Eq. (15b) is roughly larger than 7 at the same range of Q^2. See, for example, Ref. 12.

$$M_2(n, Q^2) \sim B(n+1, d(Q^2)) = \int_0^1 dx\, x^n (1-x)^{d(Q^2)-1}, \tag{36}$$

which indicates the following form for the asymptotic structure function,

$$F_2^{AS}(x, Q^2) \underset{Q^2\to\infty}{\sim} (1-x)^{d(Q^2)-1}, \tag{37}$$

where $x=\omega^{-1}=Q^2/2m\nu$. As was noted in sections III B), C), this structure function may be understood as the valence-quark component. On account of this point we must take notice of the fact that the distribution, Eq. (37), does not show a decreasing trend as $x\to 0$, contrary to the usual assumption made for the valence-quark distributions. This behaviour that $F_2^{AS}(x, Q^2)$ does not vanish at $x=0$ is in fact the origin of the divergence of the moment $M_2(n, Q^2)$ at $n=-1$ and, therefore, if we expect some mechanism which may cancel this divergence, as was discussed in Section III, then it may not be unexpected that the same mechanism will modify the structure function (37) so as to make it vanish at $x=0$ with some additional factors like x^η $(0<\eta\ll 1)$. In this respect, we recall the parametrization of the valence-quark distribution by Buras and Gaemers[11]

$$F_2^V(x, Q^2) \sim x^{\eta(Q^2)}(1-x)^{\xi(Q^2)-1}, \quad \eta, \xi>0, \tag{38}$$

with

$$\eta(Q^2)\to 0, \quad \xi(Q^2)\to\infty \text{ as } Q^2\to\infty. \tag{39}$$

Here we just mention that our result (37) may correspond to the asymptotic form of (38) in the limit of large Q^2 where $\eta(Q^2)$ tends to zero. In addition to the possibility that a detailed knowledge of resonances may enable us to modify the behaviour of F_2^{AS} near $x=0$, there is, on the other hand, the possibility that the resonances may only correctly build up the structure function at rather large values of x (say, $x\gtrsim 1/3$), while in the small x regions they may not reproduce the correct structure function. The latter possibility, however, seems to contradict the usual Resonance-Regge duality because we usualy believe that at small x (or large ω) the structure function may be well described by Regge poles. Our present result, at least, suggests that some examinations of the conventional assumption of the valence-quark distributions should be made, especially in the limit of large Q^2.

In Model B $(\delta=1)$, we obtain

$$M_2(n, Q^2) \sim B(n, d(Q^2)) = \int_0^1 dx\, x^{n-1}(1-x)^{d(Q^2)-1}, \tag{40}$$

or

$$F_2^{AS}(x, Q^2) \underset{Q^2\to\infty}{\sim} x^{-1}(1-x)^{d(Q^2)-1}. \tag{41}$$

This structure function may correspond to the sea-quark component, as was noted in section III B), C). This distribution (41) diverges as x^{-1} as x goes to zero, which again seems to contradict the ordinary assumption for the sea-quark distribution. The behaviour x^{-1}, in fact, might be too singular and cause the divergence of the zeroth moment of F_2^{AS}. This is completely analogous to the fact that the nonvanishing of the asymptotic structure function (37) at $x=0$ causes the divergence of the moment at $n=-1$. Therefore, as was mentioned above, in this case also we may expect that the mechanism which cancels the divergence will modify the structure function with some additional factor like $x^{\eta'}$ ($\eta'>0$). The recent analysis based on QCD[11] in fact shows the behaviour

$$F_2^S(x, Q^2) \sim x^{\eta'(Q^2)-1}(1-x)^{\xi'(Q^2)-1}, \quad \eta', \xi'>0, \tag{42}$$

with

$$\eta'(Q^2) \to 0, \quad \xi'(Q^2) \to \infty \text{ as } Q^2 \to \infty. \tag{43}$$

Note that this distribution (42) again coincides with our asymptotic distribution F_2^{AS}, (41), in the limit of $Q^2 \to \infty$.

Remembering that the present description of the scaling violation based on the resonance picture is in fact an asymptotic one, we may say that the resonance picture can describe the essential features of the patterns of scaling violation, and can also reproduce the basic requirement of the parton picture, such as the conservation of the number of the valence-quarks or of the energy-momentum tensor. This lends strong support to the correspondence hypothesis utilized throughout this paper. Therefore, we may expect a successful unification of the resonance picture with QCD or with the new scaling law.

We add one more comment about the form of the structure function. In comparing Eqs. (15a) and (15b) with Eqs. (37) and (41), respectively, we find that our result reproduce the Drell-Yan-West relation[23] in a generalized form. This may be a common feature of the resonance model.

V. Proton Form Factor Obtained from Correspondence Arguments between Resonance and Parton Pictures

It is known that the electromagnetic form factor of the proton deviates from the famous dipole form at large Q^2.[26] This deviation may be related to the scaling violation, and the discussions given in the previous sections have been done on the basis of this relation. For example, De Rújula, and Gross and Treiman[22] related the proton magnetic form factor to the structure function in QCD by assuming the Bloom-Gilman duality.[2] There resuting form factors showed consistent large-Q^2 behaviour with the data. Furthermore, the expression of the proton form factor obtained in this manner also agreed with other calculations[27] in QCD without the use of BG duality.

In the previous sections we have studied the consequences of the correspondence requirement between the resonance and the parton-field-theory pictures. In that context, if we can find expressions of resonance form factors which satisfy this requirement, we shall call such form factors as 'solutions' of the correspondence requirement. In this sense, QCD- and NSL-inspired form factors are the possible candidates for the 'solutions'. Here we study whether such form factors could describe the experimental data at large-Q^2 correctly.

By using the "form factor scaling law" $G_M(Q^2)=\mu G_E(Q^2)$ (μ is the magnetic moment of the proton), the elastic electron-proton scattering cross section in the one-photon approximation is expressed by

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{NS}\left(\frac{G_M(Q^2)}{\mu}\right)^2\left[\frac{1+\mu^2\tau}{1+\tau}+2\tau\mu^2\tan^2\left(\frac{\theta}{2}\right)\right], \tag{44}$$

where $(d\sigma/d\Omega)_{NS}$ denotes the cross section for scattering from a point proton, θ is the scattering angle in the Lab-frame and $\tau=Q^2/4m^2$ (m is the proton mass).

The form of the proton magnetic form factor considered in the present analysis is the one given by Eq. (15a), i. e.,

$$G_M(Q^2)=\mu/(1+Q^2/\lambda m^2)^{d(Q^2)/2} \tag{45}$$

where $d(Q^2)$ are given by Eqs. (20) and (26),

$$d(Q^2)=\begin{cases} A(Q^2/Q_0^2)^{\alpha} & (46a)\\ 4G\cdot\ln\left[\dfrac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]+p+1. & (46b)\end{cases}$$

In Eq. (46b) we set $p=3$ and G is given by

$$G=4/(33-2f), \tag{47}$$

where f is the number of quark-flavour. Q_0 is a reference momentum and λ, α and Λ are paramenters to be determined by fitting the data. Eq. (46a) corresponds to NSL and Eq. (46b) to QCD-type solution.

In the actual analysis we examine the ratio Q_M/G_D where G_D is the dipole formula

$$G_D(Q^2)=\mu/(1+Q^2/0.710)^2. \tag{48}$$

The above ratio is normalized to 1 at $Q^2=Q_0^2$ and we take $Q_0^2=3.759$ GeV^2.

Figs. 1a and 1b show the results of NSL, Eq. (46a). We fitted nine experimental points, ranging from $Q^2=5.075$ GeV^2 to 25.03 GeV^2. Fig. 1a shows the best fit, while Fig. 1b is for the two cases with the two specific values of λ which will be discussed later. The values of λ and α with the corresponding χ^2 are given in Table 1.

Figs. 2a and 2b show the fit for the QCD-type solution, Eq. (46b). Fig. 2a shows the result of the 4-flavour model ($f=4$ in Eq. (48)), and Fig. 2b the 6-flavour model ($f=6$). The values of λ, Λ^2, and χ^2 are given in Table 2.

Discussions on several points are in order.

i) We pointed out in the preceding sections that NSL is able to *exactly* satisfy the correspondence requirement. As can be seen in Fig. 1a, 1b and Table 1, NSL also gives excellent fits to the experimental proton form factor, and we take this as another strong support for NSL.

ii) Fig. 1b shows the results of the one-paramenter (α) fit corresponding to two input values of λ. The case of $\lambda=1$ corresponds to the original expression (Eq. 15a), and $\lambda=0.710/0.880$ to the case of $\lambda m^2=0.710$, the "dipole-mass" used in Eq. (48). The best fit value of λ is quite close to 1.

iii) The parameter α in Eq. (46a) essentially measure the magnitude of the anomalous dimension of the spin-n operator in NSL

$$\gamma(n)=\alpha(n+2). \tag{49}$$

This parameter was estimated from the scaling violation in lepton-nucleon scattering.[16),28)] The present result with $\lambda=0.710/0.880$ is roughly consistent with the value obtained by Kawaguchi and Nakkagawa,[28)] and that the results of the best fit and $\lambda=1$ give values

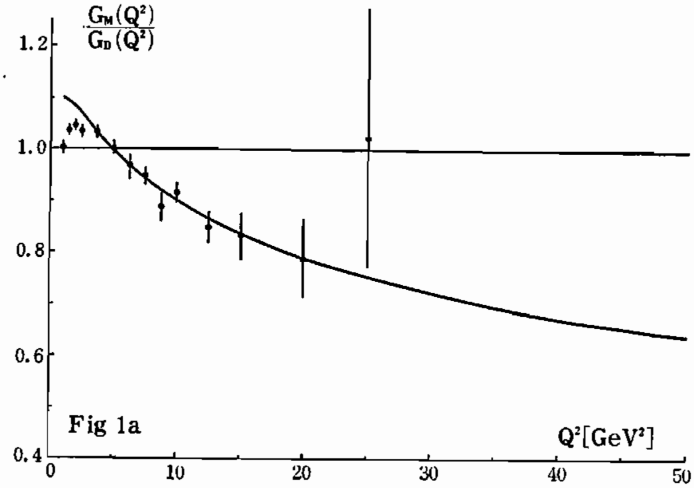

Fig. 1a The best fit result in NSL. Data are taken from Ref. 1.

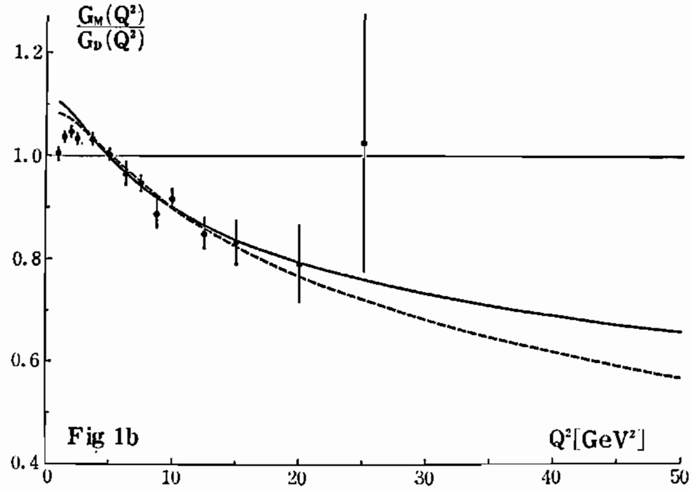

Fig. 1b Results in NSL corresponding to two input values of the parameter λ. Solid line: $\lambda=1$, dashed line: $\lambda=0.710/0.880$.

	λ	α	χ^2 (9 points)
Best Fit	0.961	0.740×10^{-2}	5.238
λ input	1.0	0.245×10^{-2}	5.258
	$\frac{0.710}{0.880}$	0.282×10^{-1}	5.598

Table 1 The values of the paramenters and χ^2 in NSL.

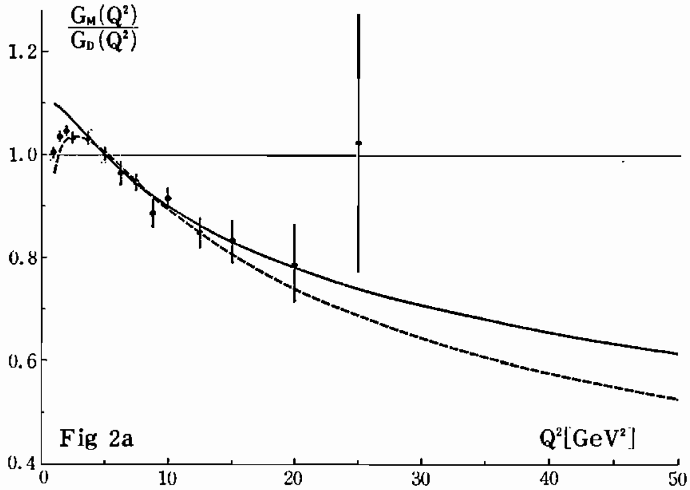

Fig. 2a Results in the four-flavour model of the QCD type solution. Solid line: best fit, dashed line: $\Lambda^2=0.2$ GeV².

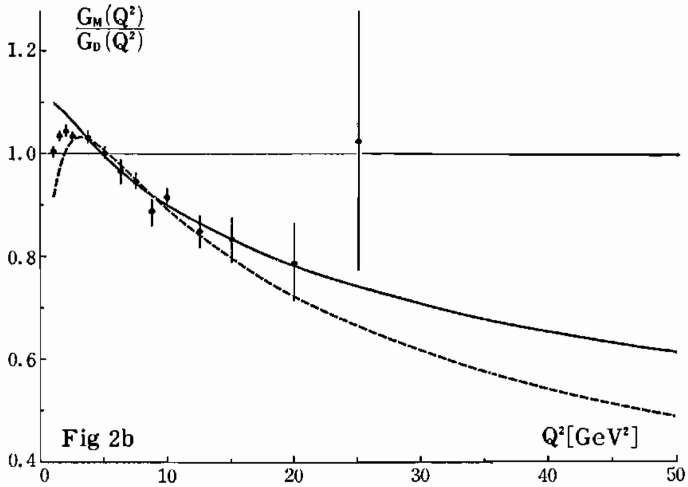

Fig. 2b Results in the six-flavour model of the QCD type solution. Solid line: best fit, dashed line: $\Lambda^2=0.2$ GeV².

		Λ^2	λ	χ^2 (9 points)
$f=4$	Best Fit	0.154×10^{-1}	0.830	5.273
	Λ^2 input	0.2	1.164	6.992
$f=6$	Best Fit	0.411×10^{-2}	0.818	5.285
	Λ^2 input	0.2	1.302	9.007

Table 2 The values of the paramenters and χ^2 in the QCD type.

smaller than those obtained from scaling violation.*)

iv) The QCD-type solution, Eq. (46b), also gives good fit to the data. An interesting feature of this model is the following: if we use $\Lambda^2=0.2$ GeV^2 suggested by the analysis of the scaling violation,[29] the theoretical curve (with λ determined by fitting data for $Q^2\geq 5.075$ GeV^2) seems to fit the small Q^2 data down to $Q^2\simeq 1.0$ GeV^2 as well, especially in the four-flavour model. This choice of the parameter Λ^2, however, gives a little faster decrease than the data in large Q^2 region.

v) The best values of Λ^2 in the QCD-type solution are significantly smaller than 0.2 GeV^2. As was mentioned earlier this may due to the fact that QCD (in the leading logarithmic approximation) does not exactly satisfy the correspondence requirement based on the Bloom-Gilman duality.

vi) Finally we should mention that both 'solutions of the correspondence requirement' (NSL and QCD-type) give excellent fits to the experimental proton form factor, despite the rather simple expressions. Furthermore, we have already known that both QCD and NSL can also describe the scaling violation. The remaining problems are a) whether these 'solutions' can give simultanious description of both the scaling violation and the form factor with the *same* values of parameters, and b) how to descriminate various 'solutions'. Partly because of the experimental uncertainty and partly because of the theoretical difficulty, we cannot at present answer these questions.

VI. Summary and Discussions

Throughout this paper we stressed the importance and the usefulness of the correspondence arguments between the resonance and the parton-field-theory pitures. In fact we have studied the behaviour of the moment sum rules and obtained the general conclusion that the conventional field theories with the ultra-violet fixed points should be ruled out as a likely candidate for the strong interactions. This conclusion may be important because, up to now, experimental results cannot conclusively decide which type of field theory is to be favoured.

More generaly the resonance picture seems to favour those theories predicting an indefinitely rising (as n tends to infinity) anomalous dimension $\gamma(n)$ of the spin-n operator. Up to now we know only two theories with such $\gamma(n)$: QCD and the new scaling law. We have noted that QCD may be compatible with the resonance picture and may be able to unify the resonance and the parton pictures. This conclusion partly confirms and gives support to the qualitative "explanation" of the Bloom-Gilman "local" duality by the asymptotic freedom. The correspondence between the resonance and QCD is, however, rather *qualitative*, namely, the predictions from QCD are not exactly the same

*) Somewhat larger values obtained in Refs. 16 and 28 are probably due to the fact that in those papers a) the contributions from valence and sea quarks were not separated and b) the data with small Q^2 (down to $Q^2\simeq 0.5$ GeV^2) were included.

as the asymptotic description based on the resonance picture.*)

We have found that NSL is also quite satisfactory: the resonance model with $d(Q^2)$ behaving as

$$d(Q^2) \sim (Q^2)^{\alpha}$$

gives exactly the same pattern of scaling violation as does NSL with anomalous dimension $\gamma(n)$,

$$\gamma(n) = \alpha(n+1), \text{ or, } \alpha n,$$

depending on the nonsinglet or the singlet operators. In this case the correspondence may be complete.

Analysis of the scaling violation based on NSL has already been done and it fits the experimental data very well. We know, however, very little about the fundamental basis of NSL. We know only the followings: a) it corresponds to the theory which predicts $\gamma(n)$ to be proportinal to n, and b) it may have its origin from the successive appearance of the new mass scals. The point b) makes us imagine the resemblance between the resonance picture and the origin of NSL. As for a), it should be noted that the linear increase of $\gamma(n)$ with n is the extreme case, allowed by the Nachtmann's positivity conditions,[30] opposite to the exact scaling.

Finally we give some discussions about the parametrization of the structure function of Ref. 11, about which we have briefly mentioned in section IV (see, Eqs. (38), (39), (42) and (43)). The results of Ref. 11 and ours agree for $Q^2 \to \infty$ as the difference is in the appearance of the exponent $\eta(Q^2)$ which goes to zero as $Q^2 \to \infty$. As was discussed earlier, the factor, x^{η} may be expected to come from the *same* mechanism which cancels the divergence of the moment at $n=-1$ (or $n=0$). We have, however, not succeeded in constructing a concrete example of such a mechanism which cancels the divergence and at the same time produces such a modification of the structure function. More careful and detailed study on this point should be made.

Meanwhile, it seems worthwhile to study the consequence of the additional factor like $x^{\eta(Q^2)}$ in the resonance model. If we set every complication aside, then by reversing the present procedures (for simplicity, we consider only the valence-quark parametrization (38)) we arrive at a modified Model A, where $G_k(Q^2)$ is now replaced by

$$G_k(Q^2) \simeq \left[\frac{\lambda}{1+\lambda}\right]^{\eta(Q^2)/2} [1+\lambda]^{-d(Q^2)/2}, \quad \lambda \equiv Q^2/M_k^2. \tag{50}$$

This is essentially the Model B with negative value of δ. In order to guarantee the Q^2-independence of the moment $M_2(n, Q^2)$ at $n=-1$, $G_k(Q^2)$ should have the extra factor $1/B(\eta(Q^2), d(Q^2))$, i.e.,

$$G_k(Q^2) \simeq \frac{1}{B(\eta(Q^2), d(Q^2))} \left[\frac{\lambda}{1+\lambda}\right]^{\eta(Q^2)/2} [1+\lambda]^{-d(Q^2)/2}. \tag{51}$$

This form factor has the same asymptotic behaviour as the original one Eq. (15), and is similar to the one used by Tajima.[4] We are not sure whether the form factor (51)

*) Remember, however, Eq. (25) and the related discussion in section III B.

gives a satisfactory description of the experimental data or not.*) On the basis of the correspondence hypothesis, however, it is certainly an interesting parametrization of the resonance form factors.

In the above considerations we have put all the effects from the additional factor $x^{\gamma(Q^2)}$ into the form of $G_k(Q^2)$. There is of cause the possibility of identifying (part of) the effects otherwise, but such a consideration will force us to go into the details of the resonances, and this is beyond the scope of our present investigation.

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*) As was studied in section V, the original form factor Eq. (15), the 'solution' of the correspondence requirement, gives excellent fit to the experimental data of the proton form factor. There would be, however, some differences between (15) and (51) in the small and intermediate regions of Q^2, and another study might be necessary in order to test the form factor expression (51).

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