Calculating the Finite-Temperature Effective Potential with the Theory Renormalized at Zero-Temperature*

Hisao NAKKAGAWA and Hiroshi YOKOTA
Institute for Natural Science, Nara University

ABSTRACT

In this paper we apply the renormalization-group (RG) inspired resummation method to the one-loop effective potential at finite temperature evaluated in the massive scalar ϕ^4 model renormalized at zero-temperature, and study whether our resummation procedure *a la* RG successfully resum the dominant correction terms apperaed in the perturbative caluculation in the $T = 0$ renormalization scheme or not. Our findings are i) that if we start from the theory renormalized at $T = 0$, then the condition that may perform the resummation of dominant corrction terms actually generates new large terms of the same order of magnitude, thus the resummation program totally breakes down, indicating ii) that the perturbative calculation with the theory renormalized at $T = 0$ does not fit for the starting basis for carrying out the resummation of temperature-dependent large corrction terms. In the theory renormalized at an arbitrary finite temperature the resummation program successfully works, carrying out the full resummation of dominant correction terms. Result in the theory renormalized at finite-temperature shows, with the one-loop knowledge alone, the second order nature of the phase transition in the model.

I. Introduction and summary

To understand the phenomena that occur in a hot and/or dense environment we must carry out the investigation with thermal field theories (field theories at finite temperature/density, hereafter we call them as TFT's). It is well known that, if the theory is renormalizable, the renormalization of the TFT can be accomplished by the renormalization of the vacuum version of the field theory, namely that there do not appear any new kind of ultraviolet divergences which depend on the temperature/density[1]. With this fact many of works invetigating the phenomena that occur in a hot environment have been carried out with TFT's renormalized at zero temperature.

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We now know that, because of the appearence of an additional quantity with large mass-scale, i.e., the temperature T of the environment, the perturbation theory of the TFT cannot be simply organized by the naive perturbative expansion in terms of the coupling g of the theory. In fact there appear large "perturbative" correction-terms having structures $(gT)^{2n}$ at the n-th order, which spoils the naive pertubative expansion. Thus in the TFT, systematic resummation, such as the hard-thermal-loop (HTL) resummation[2], of such large "perturbative" correction-terms is inevitable to get meaningful results. The perturbation theory of the TFT must be the perturbation theory of the effective theory constructed after the systematic resummation. The HTL-resummed effective theory may have some "double-counting" troubles because of the introduction of the "separating" scale. Thus we should try further to exploit another resummation procedures that enable us to construct the perturbation theory of the resulting effective theory more straightforwardly.

Recently we have proposed a new systematic resummation procedure inspired by the renormalization-group (RG) improvement[3],[4]. This procedure is nothing but the RG improvement of the theory, and the perturbative calculation after the resummation, namely after the RG improvement, has been well studied, showing there are any trouble of "double-counting" of the diagrams. Thus it may be interesting to ask the following question; In which calculational scheme the new resummation procedure can give the effective theory that can develop efficient perturbative calculations?

In this paper we present the results of application of the RG-inspired resummation to the one-loop calculation of the effective potential (EP) at finite temperature in the massive scalar ϕ^4 model renormalized at zero-temperature. Perturbative evaluation of the EP, especially at finite temperature, in terms of the loop-wise expansion, however, suffers from various troubles, e.g., unreliability of the perturbation theory[5] and the strong renormalization-scheme (RS) dependence[6]. All these troubles are due to the emergence of large perturbative correction terms (large-*log* terms in the vacuum theory, and large-T (T^2) terms in addition in the thermal theory) which depend explicitly on the RS. Thus no reliable prediction can be made without solving the problem of RS-dependence. The resummation scheme *a la* RG-improvement[3],[4] is originally proposed in order to solve this problem of RS-dependence.

With the effective potential we can get the information on the nature of the phase transition of the theory, which can be a highly non-perturbative phenomenon. Thus with the present study we can see whether our resummation procedure applied to the theory renormalized at zero-temperature successfully resum the dominant correction-terms or not. Same analyses carried out in the massive scalar ϕ^4 model renormalized at finite temperatures are given elsewhere[7],[8].

Results of the present analysis can be summarized as follows;

i) In our RG-inspired resummation scheme there is a clear distinction between theories renormalized at zero-temperature and at finite temperatures. If we start from the theory renormalized at zero-temperature, then the condition that may perform the resummation of dominant large correction-terms of $(g^2T^2)^n$ actually generates new large terms of the same order of magnitude, thus the resummation scheme totally breaks down. In the theory renormalized at an arbitrary finite temperature T_0 then the resummation program succseccfully works, carrying out the full resummation of terms of order $(g^2T^2)^n$ and of order $(g^2T)^n$. In the theory renormalized at the temperature of the emvironment T the resummation program also works, giving almost the same result as at an arbitrary finite temperature T_0. In this sense our resummation procedure gives a stable conclusion when applied to the theory renormalized at nonzero finite temperature $T_0 \neq 0$.

ii) The RG-improvement of the result in the $T = 0$ renormalization at the one-loop level predicts the first order nature of the phase transition, which cannnot be trusted because of the break-down of the resummation program explained above.

iii) The RG-improvement of the one-loop effective potential of the model renormalized at nonzero finite temperatures $T_0 \neq 0$ predicts the explicit second order nature of the temperature-dependent phase transition. This result is quite stable and agrees with those in the non-perturbative analyses. We can conclude that our resummation scheme *a la* RG-improvement can be a powerful resummation method when applied to theories renormalized at nonzero finite temperature $T_0 \neq 0$.

II. Resummation procedure *a la* RG improvement

Let us focus on the massive self-coupled scalar ϕ^4 model at finite temperature,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 - hm^4, \quad (m^2 < 0)\ , \ \lambda \equiv g^2\ , \tag{1}$$

consider the case where all the calculations are performed by employing the mass-independent RS, and the theory is renormalized at an arbitrary mass-scale μ but at the zero renormalization-temperature, $T_0 = 0$. Then the key idea to resolve the RS-ambiguity is to use the fact that the exact EP satisfies a renormalization group equation (RGE),

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial\lambda} - m^2\theta\frac{\partial}{\partial m^2} - \phi\gamma\frac{\partial}{\partial\phi} + \beta_h\frac{\partial}{\partial h}\right)V = 0\ , \tag{2}$$

whose solution is

$$V\left(\phi, \lambda, m^2, h, T; \mu^2,\right) = \bar{V}\left(\bar{\phi}(t), \bar{\lambda}(t), \bar{m}^2(t), \bar{h}(t), T; \bar{\mu}^2 = \mu^2 e^{2t}\right), \tag{3}$$

where the barred quantities $\bar{\phi}$, $\bar{\lambda}$ etc. are running parameters whose responces to the change of t are determined by the coefficient functions of the RGE, γ, β etc. with the boundary condition that the barred quantities reduce to the unbarred parameters at

$t=0$. Thus, *the EP is completely determined once its function form is known at certain value of t. The problem of resolving the RS-dependence of the EP now reduces the one how can we determine, with the limited knowledge of the L-loop calculation, the function form of the EP.*

Let us notice here that in the scalar ϕ^4 model (at least in the $O(N)$ symmetric model in $N \to \infty$) the dominant large corrections appear as a power function of the effe ctive variable τ

$$\begin{aligned} \tau &\equiv \lambda\Delta_1 , \\ M^2\Delta_1 &\equiv \frac{1}{2}\sum\!\!\!\!\!\!\int \frac{1}{k^2-M^2} + (\text{one-loop counter term}), \end{aligned} \tag{4}$$

which is nothing but the renormalized one-loop self-energy correction, having the high temperature behavior $\tau \sim \lambda(T/M)^2$, where $M^2 = m^2 + \lambda\phi^2/2$. Then we can see[3),4)] that the EP can be expressed in the power-series expansion in τ;

$$V = \frac{M^4}{\lambda}\sum_{\ell=0}^{\infty}\lambda^{\ell}\ [F_\ell(\tau) + z\delta_{\ell,0}]\ , \quad z \equiv \frac{\lambda h m^4}{M^4}, \tag{5}$$

where

$$F_\ell(\tau) \equiv \sum_{L=\ell}^{\infty} v_\ell^{(L)}\tau^{L-\ell}\ , \quad F_0(\tau) = \sum_{L=0}^{2} v_0^{(L)}\tau^L\ , \ v_0^{(L)} = 0 \text{ for } L \geq 3\ . \tag{6}$$

Then we can easily find the solution to the above posed problem; at $\tau = 0$, the "ℓth-to-leading τ" function F_ℓ is given solely in terms of the ℓ-loop level potential, $F_\ell(\tau=0) = v_\ell^{(L=\ell)}$. To be noted is that in the $O(N)$ symmetric model in the large-N limit $v_\ell^{(L=\ell)}$ is a pure constant being independent of any variables in the theory. So if we caluculated the EP to the L-loop level, then at $\tau = 0$ it already gives the function form "exact" up to "Lth-to-leading τ" rder. With the L-loop potntial at hand, the EP satisfying the RGE can be given by

$$\begin{aligned} V &= \bar{M}^4(t)\sum_{\ell=0}^{L}\bar{\lambda}^{\ell-1}(t)\left[\bar{v}_\ell^{(\ell)}(t) + \bar{z}(t)\delta_{\ell,0}\right]\Big|_{\bar{\tau}(t)=0} \\ &= V_L\ (\bar{\phi}(t), \bar{\lambda}(t), \bar{m}^2(t), \bar{h}(t), T; \mu^2 e^{2t})\big|_{\bar{\tau}(t)=0}\ , \end{aligned} \tag{7}$$

where the barred quantities should be evaluated at such a t satisfying $\bar{\tau}(t) = 0$.

III. Phase structure of the simple massive scalar ϕ^4 model at $T \neq 0$

Now we explicitly apply the RG improvement procedure explained above to the finite temperature massive scalar ϕ^4 model renormalized at zero temperature, and study the phase structure. The perturbatively calculated one-loop EP in the $T = 0$ renormalization scheme is

$$\begin{aligned} V_1 &= \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 + hm^4 \\ &+\frac{M^4}{2\lambda}\left[\tau + \lambda\left\{-\frac{b}{4} + \frac{T^4}{\pi^2 M^4}L_0\left(\frac{T^2}{M^2}\right) - \frac{T^2}{2\pi^2 M^2}L_1\left(\frac{T^2}{M^2}\right)\right\}\right], \end{aligned} \tag{8}$$

where

$$\tau \equiv \lambda\left\{\frac{b}{2}\left(\ln\frac{M^2}{\mu^2} - 1\right) + \frac{T^2}{2\pi^2 M^2}L_1\left(\frac{T^2}{M^2}\right)\right\}, \tag{9}$$

$$L_0\left(\frac{1}{a^2}\right) \equiv \int_0^\infty k^2\,dk\,\ln[1-\exp\{-\sqrt{k^2+a^2}\}]\ ,\ L_1\left(\frac{1}{a^2}\right) \equiv \frac{\partial}{\partial a^2}L_0\left(\frac{1}{a^2}\right). \tag{10}$$

At the one-loop level RGE coefficient functions are $\gamma = 0$, $\beta = 3b\lambda^2$, $\theta = -b\lambda$, $\beta_h = b/2 - 2b\lambda$, where $b = 1/16\pi^2$. Thus the RG improvement can be performed analytically, obtaining the improved EP as

$$\begin{aligned} \bar{V}_1(t) &= \frac{1}{2}\bar{m}^2\phi^2 + \frac{1}{4!}\bar{\lambda}\phi^4 + \bar{h}\bar{m}^4 \\ &+\frac{\bar{M}^4}{2}\left[-\frac{b}{4} + \frac{T^4}{\pi^2\bar{M}^4}L_0\left(\frac{T^2}{\bar{M}^2}\right) - \frac{T^2}{2\pi^2\bar{M}^2}L_1\left(\frac{T^2}{\bar{M}^2}\right)\right] && (11) \\ &= \frac{1}{2}\bar{m}^2\phi^2 + \frac{1}{4!}\bar{\lambda}\phi^4 - \frac{\bar{m}^4}{2\bar{\lambda}} + \frac{T^2\bar{M}^2}{48} - \frac{T\bar{M}^3}{48\pi} + \cdots, && (12) \end{aligned}$$

where

$$\bar{M}^2(t) = \bar{m}^2 + \frac{1}{2}\bar{\lambda}\phi^2,\quad \bar{\lambda}(t) = \frac{\lambda}{1-3\lambda bt},\quad \bar{m}^2(t) = \frac{m^2}{(1-3\lambda bt)^{1/3}}, \tag{13}$$

and all the barred quantities should be evaluated at such a t satisfying the RS-fixing condition $\bar{\tau}(t) = 0$, which gives the mass gap equation[9)]

$$\begin{aligned} M^2 &= m^2 + f(\bar{M}^2)\bar{M}^2 - f(\bar{M}^2)^{2/3}m^2\ , && (14) \\ f(\bar{M}^2) &= 1 - 3\lambda bt && (15) \\ &= 1 - 3\lambda\left[\frac{T^2}{24\bar{M}^2} - \frac{T}{8\pi\bar{M}} + b\left\{\ln(\frac{4\pi T}{\mu}) - \gamma_E\right\} + \cdots\right]. && (16) \end{aligned}$$

The stationary condition $d\bar{V}_1/d\phi = 0$ gives

$$\frac{d\bar{V}_1}{d\phi} = \phi\left(\bar{M}^2 - \frac{1}{3}\bar{\lambda}\phi^2\right) = 0\ . \tag{17}$$

With the RG improved formula (11)~(17) in hand we can study the phase structure of the model.

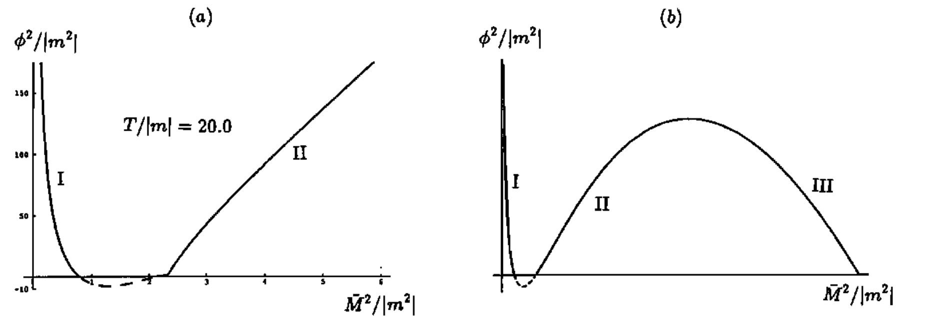

Figure 1: ϕ^2-$\bar{M}^2$ relations: (a) from the High temperature expression of the mass gap equation, $\bar{\tau}(t) = 0$, Eqs.(14)~(16), (b) from the "exact" mass gap equation, $\bar{\tau}(t) = 0$, Eqs.(14)~(15).

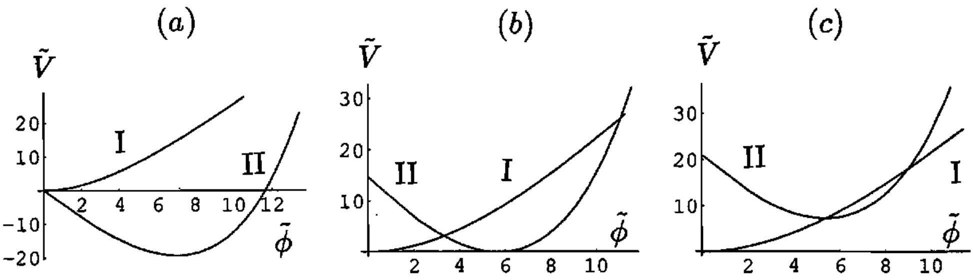

Figure 2: RG improved effective potentials at three temperatures: (a) $\tilde{T}_1 = 20.0$, (b) $\tilde{T}_2 = 23.9$, and (c) $\tilde{T}_3 = 25.0$. $\tilde{V} \equiv \bar{V}_1(\phi) - min\{\bar{V}_1(\phi = 0)\}$, $\tilde{T} \equiv T/|m|$, and the coupling is set to $\lambda = 1/20$.

III-a. High temperature expansion analysis

First let us see the result in the high temperature expansion, Eq.(12). In this case the mass gap equation, or the RS-fixing condition, Eqs. (14) and (16), gives at sufficiently high temperature the ϕ^2-$\bar{M}^2$ relation shown in Fig.1(a), indicating the exsistence of two phases I and II. The phase I is the symmetric phase and the phase II is the broken phase, see Fig.2. Also we can see is that at low temperature below T_1 the broken phase realizes the true vacuum, but that as the temperature becomes higher the symmetric and the broken phases eventually becomes mixed up thus showing the bump structure in the effective potential, and finally that at high temperature above T_3 the symmetric phase realizes the true vacuum. Phase transition in this case is strongly the first order. The broken phase II is the ordinary phase being related to the tree EP, while the symmetric phase I is generated by the resummation effect of the large-T (T^2) correction terms.

It is worth noticing that in both phases I and II the running parameters $\bar{\lambda}$ and $\bar{m}^2$ may show some peculiar behaviors, e.g., in the small ϕ region they blow up, and in the phase I $\bar{\lambda}$ becomes negative. This can be seen with the help of Eqs.(13) and (15), by noticing the fact that ϕ can be expressed as a function of $\bar{M}^2$ as

$$\phi = f^{1/3}\left[\frac{2(\bar{M}^2 f^{1/3} + |m^2|)}{\lambda}\right]^{1/2}, \tag{18}$$

where $f = f(\bar{M}^2)$, Eq. (15). Thus ϕ can become small when f becomes small (in the phase II) or when $\bar{M}^2 f^{1/3} + |m^2|$ becomes small (in the phase I). However, these are not the real trouble because if we correctly define the effective coupling and the effective mass-squared by

$$\lambda_{eff} = \frac{d^4\bar{V}_1}{d\phi^4}, \qquad m^2_{eff} = \frac{d^2\bar{V}_1}{d\phi^2}, \tag{19}$$

then λ_{eff} and m^2_{eff} show moderate behavior to be consistent with the perturbative treatment, except in the very small ϕ region where λ_{eff} becomes negative. For example, in the phase II the behavior of the effective coupling λ_{eff} at small ϕ, or at small f, can be calculated as

$$\begin{aligned} \lambda_{eff} &= \bar{\lambda}\left[-8\left(\frac{\bar{M}^2}{\bar{\lambda}\phi^2}\right)^3 + 57\left(\frac{\bar{M}^2}{\bar{\lambda}\phi^2}\right)^4 + \cdots\right] && (20)\\ &= \lambda\left(\frac{\bar{M}^2}{|m^2|}\right)^3\left[-1 + \frac{105}{16}\frac{\bar{M}^2 f^{1/3}}{|m^2|} + \cdots\right], \\ \frac{\bar{M}^2}{\bar{\lambda}\phi^2} &= \frac{\bar{M}^2 f^{1/3}}{2(|m^2| + \bar{M}^2 f^{1/3})} = \frac{\bar{M}^2 f^{1/3}}{2|m^2|}\left[1 - \frac{\bar{M}^2 f^{1/3}}{|m^2|} + \cdots\right], && (21) \end{aligned}$$

showing that λ_{eff} is small and positive except in the very small ϕ (or, small f) region. This result may have a relation with the small ϕ problem pointed out by Amelino-Camelia[10)].

III-b. Full analysis with RG improved V_1 and τ

The "exact" mass gap equation $\bar{\tau}(t) = 0$ gives the ϕ^2-$\bar{M}^2$ relation shown in Fig.1(b), indicating the exsistence of three phases: two of them, i.e., phases I and II are those already appeared in the high temperature expansion analysis, whereas the third phase III is totally new. In this phase the effective coupling λ_{eff} may become strong and the effective mass-squared also becomes very heavy, indicating this phase to be almost temperature independent super massive strong coupling phase. It is difficult to make full analyses of other properties of the third phase III because of its super strong and super massive nature.

IV. Can we rely on the results in the $T=0$ renormalization scheme?

Now let us study whether we can trust the results of the RG-inspired resummation method applied to the one-loop EP caluculated in the $T=0$ renormalization scheme. Almost all of the existing analyses in the $T=0$ renormalization scheme predict the first order nature of the phase transition, thus in this sense our results in the previous section can not be accused of this point.

The point to be examined is whether the resummation of large correction terms are consistenly carried out or not. Thus we should study the condition that may carry out the resummation, i.e., the RS-fixing condition $\bar{\tau}(t)=0$, or the mass-gap equation Eqs.(14)∼(16), giving

$$0 = \ln\frac{\bar{M}^2}{\bar{\mu}^2} - 1 + \frac{T^2}{b\pi^2\bar{M}^2}L_1\left(\frac{T^2}{\bar{M}^2}\right) \quad (22)$$

$$\sim 2\ln\frac{4\pi T}{\bar{\mu}} + \frac{4\pi^2}{3}\frac{T^2}{\bar{M}^2} - \frac{4\pi T}{\bar{M}}. \quad (23)$$

Remembering $\bar{M}^2 \sim (1/24)\bar{\lambda}T^2$, then Eq.(23) determines the RS-parameter $\bar{\mu}$ as

$$\bar{\mu} \sim 4\pi T\exp(16\pi/\bar{\lambda}), \quad (24)$$

showing that the remaining (un-resummed) $O(\lambda\ln(T/\bar{\mu}))$ terms are actually large enough to be comparable with the $O(\lambda T^2/\bar{M}^2)$ contributions. This fact indicates the breakdown of the resummation method *a la* RG in the $T=0$ renormalization with the use of RS-fixing condition $\bar{\tau}(t)=0$, showing that the results in the previous chapter can not represent the result of correct resummation of the large correction terms, thus can not be trusted at all.

Such a trouble never happens in the finite temperature renormalizations, giving a stable results showing the second order nature of the phase transition[7),8)]. It is worth mentioning that, in the large-N limit of the $O(N)$ symmetric model, after setting $\bar{\tau}(t)=0$ there appear no remaining (un-resummed) $O(\lambda\ln T/\bar{\mu})$ terms, see Eqs.(5) and (6), assuring the absence of the above trouble in the $T=0$ renormalization.

V. Discussion and comments

In this paper we investigated the results of the application of the new resummation method inspired by the renormalization-group improvement to the one-loop effective potential in the massive scalar ϕ^4 model calculated in the $T=0$ renormalization scheme. Our conclusion is that in this case the condition that may ensure the resummation of large temperature-dependent correction terms actually generates new large terms, thus the whole procedure of the resummation might get into trouble. Thus we can say that the perturbative calculation with the theory renormalized at $T=0$ does not seem to fit for the starting basis for carrying out the resummation of temperature-dependent large correction terms, at least in the present resummation procedure *a la* RG-improvement.

Discussion of the results and several comments are in order.

i) One of the reason why the problem discussed above arises may come from the fact that in the simple massive ϕ^4 model the EP can not be expressed in the simple power-series expansion in τ, Eqs.(5) and (6), but can actually be expressed as a double power-series expansion in the two effective variables τ and κ, κ being nothing but the renormalized one-loop correction to the coupling, see Refs.7), 8). Thus, as was mentioned in the last sec. IV, in the $O(N)$ symmetric model in the large-N limit where the EP can be fully expressed in the simple power-series expansion in τ, even with the problem pointed out in sec. IV, the resummation program works successfully.

The same analyses carried out in the finite-temperature renormalization schemes can show[7),8)] the following observations.

ii) Starting the perturbative calculations with the theory renormalized at an arbitrary mass-scale μ and at an arbitrary temperature T_0, we can in principle fully resum terms of $O(\lambda T/\mu)$ together with terms of $O(\lambda(T/\mu)^2)$. The key idea is to fix the arbitrary RS-parameters so as to make both of one-loop radiative corrections to the mass as well as to the coupling vanish, thus absorbing completely those terms of $O(\lambda(T/\mu)^2)$ and of $O(\lambda T/\mu)$. With the use of approximate solutions to the RGE's for the running mass-squared we can carry out the resummation program analytically, showing that the temperature-dependent transition between the symmetry-broken phase and the symmetry-restored phase proceeds through the second order phase transition.

iii) We can firstly renormalize the theory at the temperature of the environment T. In this case $O(\lambda(T/\mu)^2)$-term resummation, thus the so-called hard-thermal-loop resummation in this model, can be simply completed through the T-renormalization itself. With the lack of freedom we can set only one RS-fixing condition to absorb the large terms of $O(\lambda T/\mu)$, thus only the partial resummation of these terms can be carried out. Resulting phase structure of the model is, however, essentially the same as that in the T_0-renormalization above. In this sense our resummation method seem to give stable results so long as the terms of $O(\lambda T/\mu)$ are systematically resummed.

Finally it is worth noticing that the renormalization-group analysis of the massive scalar ϕ^4 model $T \neq 0$ has been done in great details by M. van Eijck[11)]. In his work the importance of the finite-temperature renormalization is clearly shown. However, the resummation of large temperature-dependent correction-terms is not the main issue of his work, thus no idea is given on the effective resummation-procedure of such terms.

Acknowledgments

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