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標記研究課題の下に数本の論文を作成し，その一部は既に奈良大学紀要２６号を含めた学術雑誌に発表した．この概要報告書は，研究のまとめとして作成し学術雑誌に発表予定の論文のタイトルとアブストラクトに，必要な文献を付したものである．

Improving the Effective Potential in the Massive ϕ^4 Models
— Phase Structures at Zero and at Finite Temperatures —

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Abstract

To investigate the phase structure of relativistic quantum field theory, the effective potential (EP) is widely used as a powerful and convenient tool[1)]. In calculating the EP perturbatively in terms of the loop-wise expansion it is necessary to renormalize the theory with a definite renormalization-scheme (RS). It is now well known that the perturbatively calculated EP has a strong dependence on the artificially chosen RS[2)], e.g., the renormalization scale μ and the renormalization temperature T_0 as well in thermal field theories, thus no reliable prediction can be made without resolving the problem of RS-dependence. There are another big troubles especially in thermal field theories, e.g., the unreliability of the perturbation theory[3)]. All these troubles have essentially the same origin: the emergence of large perturbative correction terms (large-*log* terms in the vacuum theory, and large-$T(T^2)$ terms in addition in the thermal theory) which depend explicitly on the RS chosen. Taking this fact into account, to break a way out of the above troubles we need some procedures to carry out systematic resummation of at least the dominant large correction terms. If we can construct such a systematic resummation procedure, then we may have some hope that it can also work as a calculational procedure for incorpolating the essential non-perturbative effect into the EP, thus helping us to understand the phase structure of the theory, toward which a variety of methods has been used[4)-6)].

Recently a simple but efficient systematic resummation scheme on the basis of the

renormalization group (RG) technique is proposed.[7),8)] Bando et al[7)] have proposed a large-log resummation scheme in the vacuum theory, and we have slightly extended their idea proposing[8)] a resummation scheme of the dominant large correction terms in the thermal case as well. In this paper we apply our resummation scheme *a la* RG, namely the RG improvmmnt procedure to the massive scalar ϕ^4 model and fully investigate the phase structure of the model both at zero and at finite temperatures[9)], showing that our RG-improvement procedure not only resolves the problem of the RS-dependence, but also incorpolates important non-perturbative effects.

Main outcomes of the present analysis are the followings;

i) At finite temperature the simple massive scalar ϕ^4 model actually has three phases, one phase I in which the symmery is restored, and two phases II and III where symmetry is broken, only one of them (the phase II) can be seen in the ordinary perturbative analysis. Other two phases I and III emerge as a result of the systematic resummation of dominant large correction terms, one of them (the phase III) having a truly non-perturbative nature and unable to be seen in the high temperature expansion analysis. Temperature-dependent phase transition proceeds through the explicit first order transition between the ordinary symmetry-broken phase II and the new symmetry-restored phase I. The small ϕ region problem, pointed out by Amelino-Camelia[10)], of the present analysis is also carefully investigated.

ii) The $O(N)$ symmetric massive scalar ϕ^4 model at finite temperature in the large-N limit has a completely different phase structure from the simple model. It has two phases: the ordinary phase and the truly non-perturbative phase. In the ordinary phase the potential changes its form as the temperature increases from the symmetry-broken wine-bottle form to the symmetry-restored one through the second order transition.

iii) The phase structure of the model at zero temperature could be studied in two different ways: studying the model at exact zero temperature by applying the resummation scheme in the vacuum theory, or studying the $T \to 0$ limit of the model at $T \neq 0$ by applying the resummation scheme in the thermal theory. We show that the simple massive scalar ϕ^4 model has a rich three-phase structure with, even at $T=0$, a phase in which the symmtry is restored. In addition, the $T \to 0$ limit of the simple model at $T \neq 0$ does not in general coincide with the same model in the vacuum theory: a) The model in the vacuum theory, i.e., at exact zero temperature has an unstable three-phase structure with one phase (i.e., the symmetric phase appearing as a result of resummation) characterized by the potential unbounded from below. Other two phases are the symmetry-broken phases. All the three phases are connected analytically, i.e., among them we can move from one phase to another with the continuous change of the parameter in the theory, thus suggesting the model being unable to exist as a stable theory. b) The $T \to 0$ limit of the same model at $T \neq 0$ is not unique, namely the model at $T \neq 0$ is not analytic at $T=0$. With a suitable $T \to 0$ limitting procedure, the $T \to 0$ limit of the model at $T \neq 0$ has, together with the three phases appeared in the model at exact zero temperature, an *isolated* new phase which can not be connected with other three phases analytically by the continuous change of the parameter in the theory. This isolated phase ia a completely massless phase with the symmetry restored. There is a $T \to 0$ procedure with which the $T \to 0$ limit of the model at $T \neq 0$ coincides with the model in the vacuum theory above in a).

iv) The $O(N)$ symmetric massive scalar ϕ^4 model in the large-N limit has a unique zero temperature limit, namely the $T \to 0$ limit of the model at $T \neq 0$ always coincides with the same model in the vacuum theory. In this model the stable two-phase structure at $T \neq 0$ survives at zero temperature and the symmetry-restored phase never appears at $T=0$. The true vacuum is realized at the minimum of the EP in the symmetry broken ordinary phase.

References

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