熱ケージ場理論の相構造の解析的研究

Chiral Phase Transitions in QED at Finite Temperature: Dyson-Schwinger Equation Analysis in the Real Time Hard-Thermal-Loop Approximation

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Although lots of efforts have been made to undestand the temperature- and/or density-dependent phase transition in thermal QCD/QED, we cannot have yet truly understood even the relation between the chiral transition and the confinement-deconfinement transition. Beginning of the relativistic heavy ion collision experiments at BNL-RHIC has attracted an increasing interest in studying the physics in thermal QCD, thus has given us an encouraging time to proceed to further investigations of the mechanism of phase transition in hot and dense gauge theories, especially in QCD and QED.

The Dyson-Schwinger (DS) equation is proven to be a powerful tool to investigate *with the analytic procedure* the phase structure of gauge theories, especially in the vacuum gauge theories [1,2,3]. However, we cannot say that, at finite temperature and/or density, the DS equation analyses of chiral and/or di-quark condensation have been carried out successfully.

In the preceding DS equation analyses [4-8], the lessons from vacuum theories have been so simply applied to thermal theories without close examination. In most analyses the ladder approximation was used by simply neglecting all the HTL effects [5,6,7], or only by taking the improper HTL effects into the gauge boson propagator [8]. As a result they have missed the essential contribution of thermal gauge field theories, especially the important effect from the "dynamically screened" magnetic mode (having in general a momentum-dependent "mass", though being massless in the static limit). Many analyses, by fixing in the Landau gauge, ignored the fermion wave-function renormalization constants (WFRCs) by taking their naive tree values [5,6,7]. Furthermore, most analyses done in the real time formalism did not discuss the physical fermion mass function Σ_R itself of the retarded propagator [7,8], with the

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neglection of its imaginary parts, together with the inaccurate use of the instantaneous exchange (IE) approximation to the gauge boson propagation [7,8]. All such improper approximation methods have caused the neglection of would-be-large contributions to the DS equation otherwise exsisted.

Then we should seriously ask whether we could rely on the previous results of the DS equation analysis on the chiral phase transition as the real consequences of thermal gauge field theories. Considering the troubles in the previous analyses [4-8] mentioned above, we should make a re-analysis by studying the hard-thermal-loop (HTL) resummed DS equation in the real time formalism, thus might giving a new understanding on the phase structure and the mechanism of phase transition in thermal gauge theories.

Main interest of the present investigation lies in clarifying what are the essential temperature effects that govern the phase transition and also in finding how we can closely take these effects into the "kernel" of the DS equation. Essential procedures of our analysis can be summerised as follows;

i) Firstly we use the real time closed-time-path (RT-CTP) formalism [9], and study the physical mass function Σ_R itself, not the Σ_{11}, of the retarded fermion propagator, because we are interested in both the real and imaginary parts.

ii) Secondly we accurately take into our analysis the fact that Σ_R is the mass function of "unstable" quasi-particle in thermal field theories, thus having non-trivial imaginary parts as well as non-trivial WFRCs. Neglection of imaginary parts and non-trivial WFRCs actually give constraint equations to be solved simultaneously, totally dismissed in the preceding analyses.

iii) Thirdly and most importantly, devoting our attention to closely estimating the dominant temperature-dependent contributions, we focus on studying the DS equation being exact up to HTL approximation: Both the gauge boson propagators and the vertex functions are determined within the HTL resummation [10,11,12], with which the gauge invariance of the result at least in the perturbative analysis is guaranteed. With the HTL resummed vertex functions [12] we can explicitly write down the HTL resummed DS equation.

iv) Finally, the gauge-paremeter dependent contribution must be carefully studied without fixing the gauge into some definite ones, such as the Landau gauge.

The third point listed above is better to be taken step by step into the actual analysis of the DS equation. In the present analysis we present the result of our first step investigation in strong QED; focussing on what happens when we take into account exactly at least the HTL resummed gauge boson propagators. Analysis in QCD and effects of fully including the HTL resummed vertices will be presented in the separate paper [13].

The DS equation for the physical, i.e., the retarded fermion self-energy function Σ_R in the

HTL approximation can be obtained by applying the following approximation to the full DS equation;

i) replace the full gauge boson propagator with the HTL resummed propagator, and

ii) approximate the full vertex functions to the HTL resummed vertex functions.

Then in the RT-CTP formalism we get in QED the desired DS equation [12]. At zero temperature, the wave function renormalization constant A(P) coincides with B(P) and equals to unity in the Landau gauge, while at finite temperature it is not. Appearence of the HTL resummed vertex functions together with the HTL resummed gauge boson propagators assures that the HTL approximation is consistently carried out in studying the HTL resummed DS equation [12], and guarantees the result being gauge invariant, at least, in the effective perturbation regime. Neglection of the HTL contribution to the vertex function, $\delta^{*}\Gamma^{\mu}{}_{\alpha\beta\gamma}$, simply brings us to the ladder DS equation with the HTL resummed gauge boson propagator. It significantly simplifies the structure of the DS equation to be examined, thus reducing the technical difficulty to handle the DS equation itself. The price to pay is to lose the assurence of gauge invariance of the results.

In the present analysis as already mentioned above, we investigate the consequences of the ladder (point vertex) DS equation with the HTL resummed gauge boson propagator. The DS equation obtained, is still quite tough to be attacked, forcing us further approximations for the analysis to be effectively carried out. However, the approximation made use of must be consistent with the HTL approximation, without missing the important thermal effects out of the kernel of the DS equation.

Here it is worth noticing that the instantaneous exchange (IE) approximation frequently used in the preceding analyses [6,7,8] is *not compatible* with the HTL approximation in the strict sense. In the exact IE-limit the HTL resummed transeverse mass function vanishes and the transeverse (magnetic) mode becomes totally massless. Namely the IE approximation discards the important thermal effect coming from the Landau damping, thus dismissing the dynamical screening of the magnetic mode. This causes the famous quadratic divergence of the Rutherford scattering cross section. The reason why in the previous analyses this divergence did not appear, is that the imaginary part of Σ_R is completely neglected there from the beginning, namely that the equation for $\mathrm{Im}\,\Sigma_R$ is totally discarded.

Taking the above into account, the approximation we further make use of is the improved IE approximation to the longitudinal gauge boson propagator, by keeping the exact HTL resummed transeverse propagator. In the IE-limit the HTL resummed longitudinal mass function. ${}^{*}\Pi^{R}{}_{L}(K)$, has a definite thermal mass $mg^{2} \sim (gT)^{2}$, representing the Debye screening due to thermal fluctuation, thus even in the IE limit the longitudinal mode can take into acount the essential thermal effect. In the present analysis the gauge is fixed to the Landau gauge ($\xi=0$).

It is fair to note that in the point vertex ladder approximation, as already mentioned above, the gauge invariance of the results is spoiled. To maximally respect the gauge invariance, we should solve the DS equations with the constraint $A(P)=1$, which guarantees $Z_2=1$, being consistent with the Ward identity $Z_1=Z_2$. This can be done [16] by successively adjusting the gauge-parameter ξ in solving the DS equations.

Now we should solve numerically the DS equations with the IE approximation to the longitudinal mode [17]. Result of the present analysis shows the two facts; i) The chiral phase transition is of second order, since a fermion mass is generated at a critical value of the temperature T or at the critical coupling constant α without any discontinuity, and ii) the critical temperature T_C at fixed value of α is significantly lower than the previous results [6,7,8], namely the restoration of chiral symmetry occurs at lower temperature than previously expected. The second fact shows that in the previous analyses the important temperature effects are neglected due to the inapropriate approximations.

The critical coupling constant α_C as a function of T, and the critical temperature T_C as a function of α, can be also determined. From these results we can estimate the critical coupling constant α_C in the limit $T \to 0$, $\alpha_C(T \to 0)$, and the value of coupling constant α where the critical temperature T_C becomes zero, $\alpha(T_C=0)$. Our result shows that, as T becomes smaller, the critical coupling constant α_C also becomes smaller and seems to consistently decrease from above to the zero temperature result. However, the estimated values, $\alpha_C(T \to 0) \sim 2.5$ and $\alpha(T_C=0) \sim 2.5$, are significantly larger than the value $\alpha_C(T=0)=\pi/3$ determined by theoretical analyses [1] of the DS equation for the fermion self-energy part $\Sigma(P)$ at zero temperature, T= 0, in the ladder approximation in the Landau gauge with the tree level photon propagator. Since the HTL resummed gauge boson propagator ${}^{*}G^{\mu\nu}$ is simply reduced to the tree level photon propagator in the limit $T \to 0$, we are not quite sure why such difference emerges.

Present result shows the correctness of our research-strategy, namely, the importance of the full HTL resummed DS equation analysis of the chiral phase transition at finite temperature/density. Further investigation along the line of our strategy is needed to answer the question how we can closely take the essential thermal effects into the "kernel" of the DS equation, which is now under investigation.

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