Consistency Condition of Perturbation Theory and Renormalization-Scheme Dependence of Perturbative QCD Calculations

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Abstract

We studied the consequences of the consistency condition imposed by the truncated perturbation theory on the renormalization-scheme (RS) dependence of perturbative QCD calculations. By studying several physical quantities being free from or insensitive to the factorization-scheme dependence, we found the followings : (1) The significance of the RS-dependence is almost independent of the process considered, (2) higher order contributions are always important and non-negligible from the point of theoretical consistency, and (3) the resolution of the RS-dependence is really important in order to make quantitative confrontations of perturbative QCD with experiment.

I Introduction

What troubles us in applying perturbative quantum chromodynamics (QCD) to some processes is the renormalization-scheme (RS) dependence[1)-11)], which comes from the arbitrariness in renormalizing the strong coupling $a=g^2/4\pi^2$. This problem has been studied extensively so far[1)-10)] mainly from the point of getting a reliable perturbative prediction, or of resolving the RS-ambiguity. Recently we studied[11)] the consequences of the consistency condition imposed on the RS-dependence by the truncated perturbation theory (TPT) (Hereafter we refer to this paper as I). The only process considered in I is, however, the deep inelastic lepton scattering off a target hadron. Then the definiteness of the consequences obtained in I might become obscured by the following two reasons. The first one is that the RS-dependence could be process-dependent. The second one is that perturbative QCD applied to such a process in which

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several hadrons participate suffers from the second ambiguity, the factorization scheme (FS) dependence[12)-14)], which results from the arbitrariness in factorizing the long distance part of the process. The FS-dependence has its theoretical importance as well as the phenomenological one. In fact, the investigation of the FS-dependence together with the RS-dependence revealed a new pathway to the problem of the scheme-dependences in perturbative QCD[14)].

The purpose of this paper is to study the problem of the RS-dependence almost parallel to I, but (i) more generally by considering three processes: the deep inelastic leptoproduction, the e^+e^- annihilation and the heavy quarkonium decay, and (ii) more carefully by studying quantities which might be free from or insensitive to the FS-dependence: the logarithmic derivative of the structure function moment, the Drell ratio and the ratio of the gluonic and leptonic widths of the heavy quarkonium. This generalization and refinement may allow us to get more definite and thorough consequences on the RS-dependence,

II The second order calculations and the consistency condition

We define the second order coupling a as the solution to the β-function equation truncated at the second order;

$$\mu\frac{\partial a}{\partial\mu}=-ba^2[1+ca]\equiv\beta(a), \tag{1}$$

namely[6)],

$$\tau\equiv b\log\frac{\mu}{\tilde{\Lambda}}=\int_{a(\mu)}^{\infty}\frac{dx}{bx^2(1+cx)}, \tag{2}$$

where b and c are the RS-independent constants given by

$$b=\frac{11}{2}-\frac{n_f}{3},\ c=(51-\frac{19}{3}n_f)/(22-\frac{4}{3}n_f). \tag{3}$$

The definition of the scale $\tilde{\Lambda}$ through Eq. (2) is an RS-dependent one. However, $\tilde{\Lambda}$'s in different RS's can be related *exactly* by a one-loop calculaion[3)], i.e.,

$$\tilde{\Lambda}'/\tilde{\Lambda}=\exp(v_1/b), \tag{4}$$

where v_1 is the one-loop coefficient given by

$$a'(\mu)=a(\mu)[1+v_1a(\mu)+\cdots\cdots]. \tag{5}$$

A physical quantity R which is a function of one physical variable Q, the relevant large mass-scale in the process considered, has a second order perturbative expression of the form

$$R=[a(\mu)]^N[1+r_1(Q/\mu,\ \mu/\tilde{\Lambda})a(\mu)]. \tag{6}$$

The coefficient r_1 depends on the particular RS used to renormalize the coupling a, and is process-dependent. Throughout this paper we set $\mu=Q$, and the parameter labeling RS's is the scale $\tilde{\Lambda}$ as is obvions from Eq. (2)[*)]

Here we consider the three processes separately.

*) For more detailed discussions on the parametrization of the RS-dependence and the consistency condition, see Stevenson, Ref. 6), and Nakka and gawa-Kawaguchi, Ref. 11).

(a) Deep inelastic leptoproduction (DIL)

In this process a physical quantity being insensitive to the FS-ambiguity is the logarithmic derivative of a moment $M_n(Q^2)$ of the structure function (only the flavor-nonsinglet combination is considered);

$$R^{\mathrm{DIL}} = -(8/\gamma_0{}^n)(\partial \log M_n(Q^2)/\partial \log Q^2) \tag{7}$$

which has a second order expression

$$R^{\mathrm{DIL}} = a[1 + r_1{}^n a],\ r_1{}^n = (4b/\gamma_0{}^n)\tilde{r}_1{}^n + c, \tag{8}$$

where $\gamma_0{}^n$ is the one-loop result of the anomalous dimension and is RS-independent. The second order coefficient $\tilde{r}_1{}^n$ was calculated by Bardeen *et. al.*[2] in the minimal subtraction scheme (MS)[15] and also in the modified MS ($\overline{\mathrm{MS}}$)[2].

(b) e^+e^- annihilation (EEA)

The Drell ratio $R_{e+e-} = \sigma_{had}/\sigma_{\mu\mu}$ is given in the second order calculation by

$$R_{e+e-} = (3\sum_i e_i{}^2)(1 + R^{EEA}) \tag{9}$$

$$R^{EEA} = a[1 + \gamma_1{}^{EEA} a]. \tag{10}$$

The coefficient $\gamma_1{}^{EEA}$ was calculated[16] in the MS and $\overline{\mathrm{MS}}$. In this process the mass scale Q is the total energy of the e^+e^- system, $Q = \sqrt{s}$.

(c) Decay of ortho-quarkonium (DOQ)

The ratio of the gluonic and leptonic widths, $\Gamma_g/\Gamma\mu\mu$, of heavy ortho-quarkonium is insensitive to the FS-ambiguity. Thus we define R^{DOQ} as

$$R^{DOQ} = (81\pi e_Q{}^2/10(\pi^2-9))(\alpha_{em}{}^2/\pi^3)(\Gamma_g{}^{OQ}/\Gamma\mu\mu^{OQ}), \tag{11}$$

which is given in the second order calculation by

$$R^{DOQ} = a^3[1 + r_1{}^{DOQ} a]. \tag{12}$$

In Eq. (11) e_Q is the fractional charge of the heavy quark. The coefficient $r_1{}^{DOQ}$ was calculated in the $\overline{\mathrm{MS}}$ by Mackenzie and Lepage[17]. Here the physical variable Q corresponds to the mass of the heavy quarkonium, $Q = M$, or to the average momentum-flow through a gluon, $Q = M/3$.

Now in order to study the RS-dependence of the second order approximant R, Eq. (6), we first define a base-scheme S_0 and calculate R in this scheme, $R_{S0}(Q)$. Next we study any other scheme S and calculate $R_S(Q)$ by imposing the theoretical relation between Λ's, Eq. (4). Then $R_{S0}(Q)$ and $R_S(Q)$ can differ only by a term of $O(a^{N+2})$, or[11],

$$\Delta(S,Q;S_0) \equiv |(R_S(Q) - R_{S0}(Q))/R_{S0}(Q)| \leqq \delta a^2, \tag{13}$$

where δ is a constant of $O(1)$. This inequality must hold for any pair of RS's, S_0 and S, and is the consistency condition imposed on the RS-dependence by the TPT. The consistency condition (13) imposes $\tilde{\Lambda}$ to satisfy the inequality

$$\tilde{\Lambda} \geqq \tilde{\Lambda}_{MIN} = Q\exp\{-[1/a + c\log(ca/(1+ca))]1/b\}, \tag{14}$$

where

$$a = \sqrt{\Delta(S,Q;S_0)/\delta} \tag{15}$$

III Analysis

In this analysis we consider five RS's which are familiar in the perturbative QCD calculations: MS[16], $\overline{\mathrm{MS}}$[2], the momentum-space subtraction scheme (MOM)[3),4)*)], and the two optimum schemes, one of which is based on the principle of minimal sensitivity (PMS)[6], the other on the principle of fastest apparent convergence (FAC)[5]. Data used to determine the QCD scale $\tilde{\Lambda}$ in the above three processes, DIL, EEA and DOQ, are the followings ; (a) DIL : the data for F_2^{p-n}, Q^2-values ranging from 4.0 to 25.0 GeV^2 and the moment $n=2\sim6$[18]. (b) EEA: the compilation of R values[19], and (c) DOQ : the leptonic and gluonic decay widths[20] of Υ and J/ψ, $\Gamma\mu\mu=1.16\pm0.15$ and $\Gamma_g=27\pm7$ for Υ, $\Gamma\mu\mu=4.8\pm0.6$ and $\Gamma_g=44\pm6$ for J/ψ in keV unit. The number of quark-flavors, n_f in Eq. (3), is taken to be $n_f=4$ in the DIL and DOQ, whereas $n_f=5$ in the EEA.

3-1 The determination of the QCD scale Λ

Here we consider for convenience the conventional scale Λ, as defined by Buras *et. al.*[21], which is related to $\tilde{\Lambda}$ by

$$\Lambda=(2c/b)^{c/d}\tilde{\Lambda}. \tag{16}$$

The scale Λ can be fitted to experimental data in each RS by the least χ^2-method. Results are given, together with the results obtained in I, in Table I. In the DIL errors of roughly 100～200 MeV should be understood[1], whereas in the DOQ errors are at most 50 MeV[17)**)]. In the PMS and FAC, Λ's are given provided the original calculational scheme is the $\overline{\mathrm{MS}}$. For details, see I.

Table I. The best-fit values of the QCD scale Λ. In the schemes PMS and FAC, Λ's are given provided the original calculational scheme is the $\overline{\mathrm{MS}}$. Results for M_n itself are taken from I. As for the possible errors, see Text.

	Λ^{exp} (GeV)				
	M_n	$\partial\log M_n/\partial\log Q^2$	$R_{e^+e^-}$	$\Gamma_g/\Gamma\mu\mu(\Upsilon)$	$\Gamma_g/\Gamma\mu\mu(J/\varphi)$
MS	.44	.33	.03	.07	.04
$\overline{\mathrm{MS}}$	.49	.48	.05	.11	.07
MOM	.72	.72	.09	.20	.11
PMS	.33	.30	.05	.09	.05
FAC	.34	.30	.05	.10	.05

The following fact is worth while noticing: Although the best-fit values of Λ vary considerablly process to process considered, the ratio of them among the different RS's is almost process-independent, namely

*) We only consider the momentum-space subtraction in the Landau gauge. See, Refs. 3) and 4).

**) The experimentally allowed errors to the best-fit values of Λ given in Table I can be determined in the PMS scheme as follows ; (a$_1$) DIL, M_n : $\Lambda=330^{+150}_{-190}$, (a$_2$) DIL, $\partial\log M_n/\partial\log Q^2$: $\Lambda=300^{+160}_{-220}$, (c$_1$) DOQ, Υ : $\Lambda=94^{+54}_{-51}$, (c$_2$) DOQ, J/ψ : $\Lambda=52\pm17$ in MeV unit. In the analysis of the Drell ratio errors of roughly 500 MeV could be possible.

$$\Lambda^{exp}_{MS} : \Lambda^{exp}_{\overline{MS}} : \Lambda^{exp}_{MOM} = 1:1.5:2.2 \quad \text{(DIL)}, \tag{17a}$$
$$1:2.1:3.7 \quad \text{(EEA)}, \tag{17b}$$
$$1:1.6:2.9 \quad (\text{DOQ}, \Upsilon), \tag{17c}$$
$$1:1.6:2.7 \quad (\text{DOQ}, J/\psi). \tag{17a}$$

This experimental ratio is, however, significantly different from the theoretical ratio (4)*)

$$\Lambda^{th}_{MS} : \Lambda^{th}_{\overline{MS}} : \Lambda^{th}_{MOM} = 1:2.66:5.73. \tag{18}$$

In fact, even if we take account of the reasonable errors, it remains to be difficult that the experimental ratios in the DIL and DOQ (17 a, c and d) become consistent with the theoretical one (18).

3-2 Consistency analysis

We present results of the analysis based on the consistency condition in Figs. 1~4 and in Table II. The constant δ in the inequality (13) and Eq. (15) is set to be $\delta=10$, which might show us the maximal domain of the scale Λ permitted by the consistency condition. Let us study the three processes separately.

(a) Deep inelastic leptoproduction

Figure 1 shows the Q^2-and the moment-dependences of $\Lambda_{\text{MIN}} \cong 0.89 \tilde{\Lambda}_{MIN}$ where the PMS scheme is taken as a base-scheme S_0 and other four schemes, $\overline{\text{MS}}$, MS, MOM and FAC, are studied. The input scale Λ in this case is the PMS-value of Λ, $\Lambda^{\text{exp}}_{\text{PMS}}=300$ MeV. When the MOM scheme is studied the lowest moment n=2 gives an extremely small Λ_{MIN} of $O(10^{-9})$ or less in GeV unit, and the result is not reproduced in this figure. The errors attached to the results for the lowest moments ($n=2$ in the MS, $\overline{\text{MS}}$ and FAC and $n=4$ in the MOM) correspond to the errors of Λ^{exp} (which is now taken to be ±100MeV) fitted to the experimental data, and give an idea about the definiteness of our conclusion.

We can at first easily see that Λ_{MIN}'s in the schemes FAC and MOM decrease as Q^2 increases, whereas Λ_{MIN}'s in the MS and $\overline{\text{MS}}$ increase as Q^2. Comparing Λ^{exp} in Table I with Λ_{MIN} in this figure we can conclude that the FAC and MOM are consistent with the PMS, whereas the $\overline{\text{MS}}$ and MS are not. While consistent set of RS's give equivalent perturbative results**), inconstent set of RS's give inequivalent results, thus permitting the independent confrontations with the data. It is interesting that there is a correlation between the behavior of the Q^2-dependece of Λ_{MIN} and the theoretical consistency satisfied by the RS considered.

Figure 2 shows the results when the MOM is taken as a base-scheme S_0. The result of the second moment (n=2) in the PMS is not given because of the smallness of Λ_{MIN}, which is of $O(10^{-9})$ or less in GeV unit. In Fig. 3 we present the results in the MS and MOM where the $\overline{\text{MS}}$ is the base-scheme and the

*) This is also noted by Bardeen *et al.*[2] and Haruyama *es al.*[9] in the analysis of the DIL.

**) This conclusion confirms those which claim the equivalence of the FAC and the PMS, see Refs. 10).

input Λ is the Λ^{exp} in Table I, $\Lambda_{\overline{MS}}^{exp}=480$ MeV. Results in the PMS are not reproduced because we have already had an idea about the inconsistency between the schemes $\overline{\text{MS}}$ and PMS. Those in the FAC are also not shown. Added are the results in which we used, as the input value of $\Lambda_{\overline{MS}}$, the "world $\Lambda_{\overline{MS}}$ in 1981"[1] which is to be $\Lambda_{\overline{MS}}^{1981}=160$ MeV (errors omitted). From Figs. 2 and 3 we can say that the schemes $\overline{\text{MS}}$ and MOM can not satisfy the theoretical consistency, despite of the fact that both schemes are claimed[4),7)] to give small perturbative corrections thus to give reliable perturbative predictions in the DIL. Also from Fig. 3 we can conclude that the $\overline{\text{MS}}$ and MS are inconsistent with each other. It should de noted that these conclusions are insensitive to the experimental errors, as can be seen from Fig. 1.

Results of the analyses where the schemes FAC and MS are taken as base-schemes are not reproduced simply because the FAC is almost equivalent to

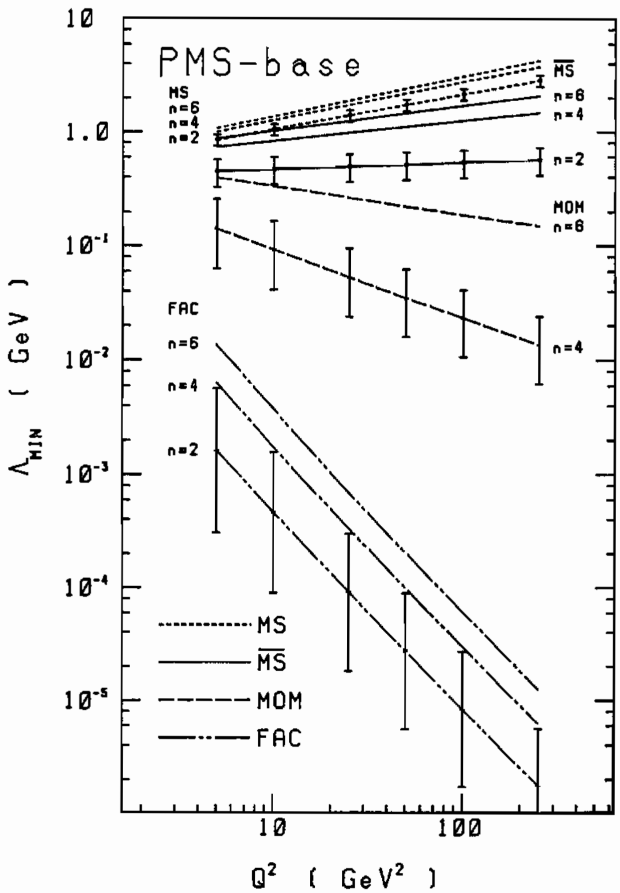

Fig. 1 Λ_{MIN} in the analysis of the DIL with the PMS as a base-scheme. Errors attached correspond to those of Λ^{exp}, see Text.

the PMS in phenomenological sense[10] and because the MS is apparently inconsistent with any other four RS's, as can be easily seen from Figs. 1~3.

(b) e^+e^- annihllation

Figure 4 shows the $\sqrt{s}$-dependences of $\Lambda_{MIN}\cong 0.87\tilde{\Lambda}_{MIN}$. The FAC-velues of Λ_{MIN} are not shown because the FAC is almost equivalent to the PMS[10]. In the $\overline{MS}$-based analysis we studied the two input values for $\Lambda^{exp}_{\overline{MS}}$ one is our best-fit value $\Lambda^{exp}_{\overline{MS}}=50$ MeV in Table I, the other the "world $\Lambda_{\overline{MS}}$ in 1981"[1] $\Lambda^{1981}_{\overline{MS}}=160$ MeV. Comparison of Λ^{exp} in Table I with Λ_{MIN} in Fig. 4 may allow us to say that the three RS's, PMS, FAC and $\overline{MS}$, satisfy the consistency condition of the TPT, and that the remaining two RS's, MOM and MS, are isolated from any other RS's. These observations, however, might be dimmed because of the large experimental uncertainties inherent to the e^+e^- annihilation process.

(c) Decay of ortho-quarkonium

We pressent in Table II the values of ΛMIN$\cong 0.89\ \tilde{\Lambda}$MIN determined through the analysis of the ratio of the gluonic and leptonic widths of Υ and J/ψ, where

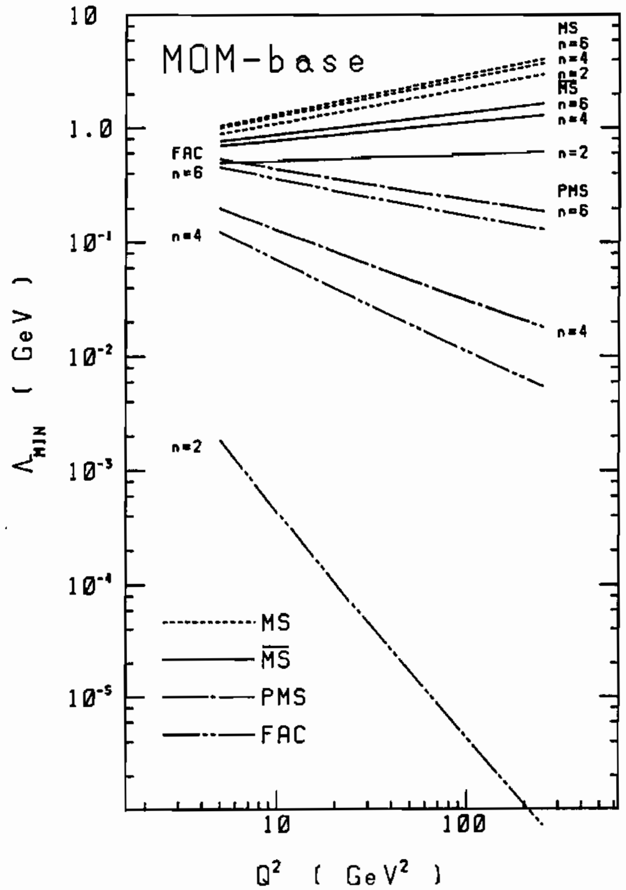

Fig. 2 Λ_{MIN} in the analysis of the DIL with the MOM as a base-scheme.

Table II. Λ_{MIN} in the analysis of the DOQ, where the renormalization scale is chosen to be the mass of the quarkonium, $\mu=M$.

		Λ_{MIN} (GeV)				
		MS	$\overline{\text{MS}}$	MOM	PMS	FAC
Υ	PMS-base	3.96	1.92	$.39\times10^{-11}$	—	$.30\times10^{-5}$
	FAC-base	3.95	1.89	$.15\times10^{-6}$	$.31\times10^{-5}$	—
	MOM-base	3.95	1.91	—	$.40\times10^{-11}$	$.15\times10^{-6}$
	$\overline{\text{MS}}$-base	3.40	—	2.30	2.31	2.27
	MS-base	—	4.36	5.29	5.29	5.28
J/ψ	PMS-base	1.44	.87	$.13\times10^{-2}$	—	$.86\times10^{-5}$
	FAC-base	1.44	.86	$.20\times10^{-3}$	$.94\times10^{-5}$	—
	MOM-base	1.43	.85	—	$.13\times10^{-2}$	$.11\times10^{-3}$
	$\overline{\text{MS}}$-base	1.24	—	1.05	1.08	1.07
	MS-base	—	1.63	1.97	1.99	1.98

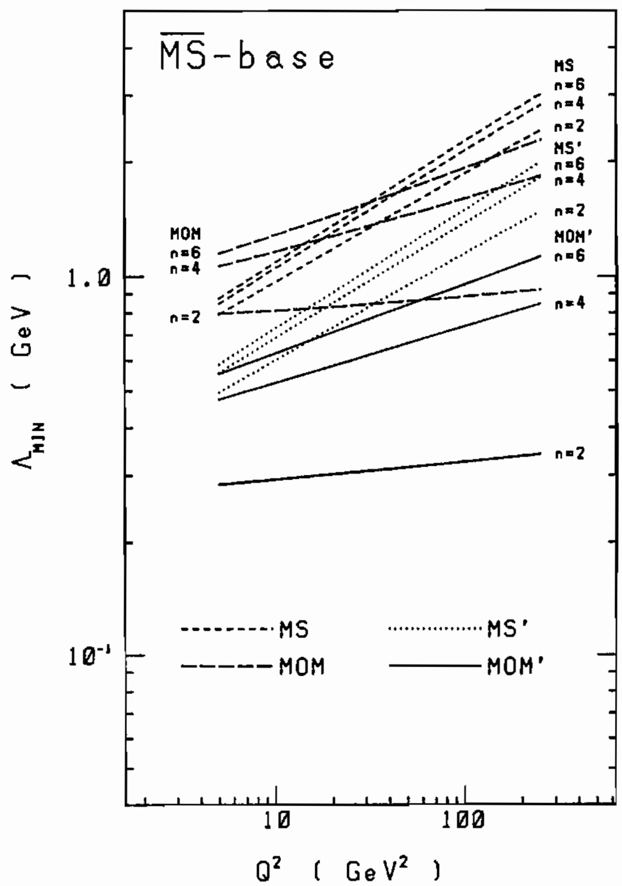

Fig. 3 Λ_{MIN} in the analysis of the DIL with the $\overline{\text{MS}}$ as a base-scheme. Results with the use of the "world $\Lambda_{\overline{MS}}$ in 1981" (which are denoted by the primed notations, MS', MOM',etc.) are shown together with those with our best-fit value of $\Lambda_{\overline{MS}}$, see Text.

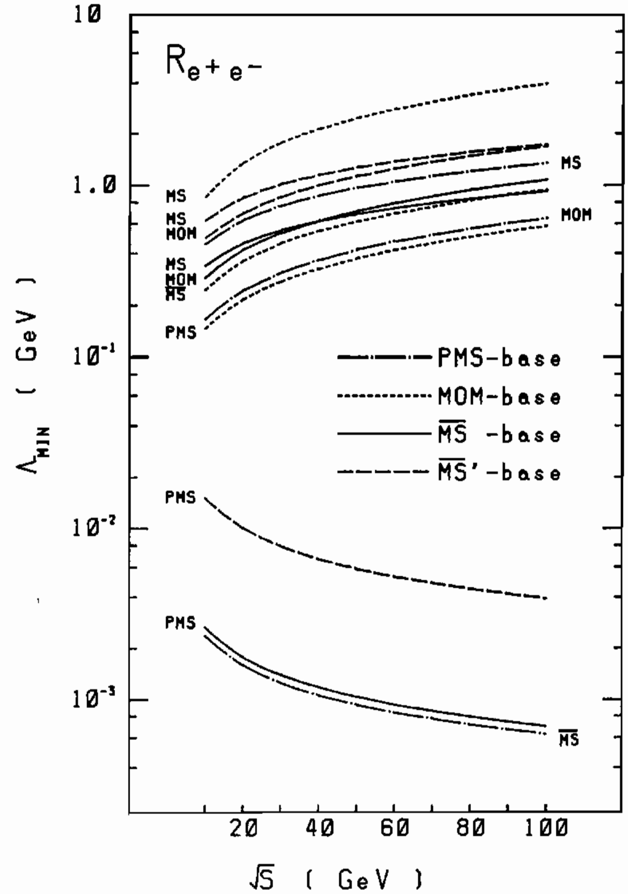

Fig. 4 Λ_{MIN} in the analysis of the EEA with the four choices of base-schemes, PMS, MOM, $\overline{MS}$ and $\overline{MS}$'. The MS' denotes the $\overline{MS}$ with the "world $\Lambda_{\overline{MS}}$ in 1981", see Text.

the renormalization scale is taken to be the mass of the quarkonium considered. From Tables I and II we can say that three RS's, PMS, FAC and MOM, can satisfy the consistency cond- ition of the TPT with each other, whereas the $\overline{MS}$ and MS are inconsistent with any other RS's. This conclusion does not depend on the specific quarkonium considered, Υ or J/ψ, and may be definite up to the experimental errors.

IV Conclusion and discussions

In this paper we analyzed in various physical processes the second order perturbative QCD calculations of quantities being free from or insensitive to the FS-dependence, and studied the consequenses of the consistency condition of the TPT imposed on the RS-dependence. As a result we confirmed the conclusions obtained in I and the importance of the resolusion of the RS-dependence : Even if we calculate higher order corrections and even when we consider large Q^2

regions, the further higher order contributions being neglected are always important and non-neglible from the point of the theoretical consistency. Although any RS is formally equivalent, only the consistent set of RS's satisfying with each other the requirement(13)can give equivalent perturbative results. Those RS's which can not satisfy the constency condition(13) give inequivalent perturbative results, thus permit the independent confrontations with experiment.

Several discussions on the present results are in order.

i) The process-independence of the experimental ratio of the QCD scale Λ, Eqs.(17), seems to be interesting. This fact indicates that the significance of the higher order contributions and of the RS-dependence is almost independent of the process considered.

ii) In the deep inelastic leptoproduction the analysis of $\partial\log M_n/\partial\log Q^2$ gives slightly improved ratio (17a), which should be compared with

$$\Lambda_{MS} : \Lambda_{\overline{MS}} : \Lambda_{MOM} = 1 : 1.1 : 1.6$$

obtained in I through the analysis of M_n itself. This result may reflect the fact that $\partial\log M_n/\partial\log Q^2$ is insensitive to the FS-dependence while M_n itself is not.

iii) The analysis of the Drell ratio seems to give different conclusion from those of other two processes. However, taking account of the large experimental uncertainties inherent to the e^+e^- annihilation process this discrepancy can not be taken so seriously.

iv) In the analysis of the heavy quarkoninm decay, the choice of the renormalization scale $\mu=M/3$ does not give us any new informations concerning the purpose of this paper, except in this choice the MOM scheme can not reproduce the experimental data[17]. As a result we do not discuss this choice separately.

v) The discrepancy between the Λ-parameters fitted to the Υ-decay and to the J/ψ-decay may have some importance with respect to the FS-dependence of the ratio $\Gamma_g/\Gamma_{\mu\mu}$ considered in this paper. Although it is usually believed[22] that the effect of the wave-function might be cancelled by taking the ratio of the gluonic and leptonic widths, the above discrepancy may force us to have some doubts about the validity of this assumption.

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