Deep Inelastic Processes and SU(4) Symmetry

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I. Introduction

Symmetry properties of the strong interaction lead to various relations among the electromagnetic and weak structure functions. They are conveniently obtained by use of the quark parton model or the light-cone algebra.[1] The validity of these symmetry relations, however, is independent of the validity of dynamical results such as the Bjorken scaling: many of them can indeed be obtained from the conventional assumptions of strong interactions (such as the absence of exotic exchanges, duality, etc.), as has been shown in the case of old currents without charm pieces[2),3)].

The purpose of this paper is to study the consequences of SU(4) symmetry[4),5)] in deep inelastic processes using the conventional ideas on two-body hadronic reactions. The electromagnetic (J^{em}) and charged weak currents ($J_{\pm}^{W}$) are taken to be[6)]

$$J^{em}=V^3+(1/\sqrt{3})\,V^8-\sqrt{2/3}V^{15}+(\sqrt{2/3})\,V^0, \tag{1a}$$

$$J_+^W=(V-A)^{1+i2}\cos\theta+(V-A)^{4+i5}\sin\theta$$
$$-(V-A)^{11-i12}\sin\theta+(V-A)^{13-i14}\cos\theta, \tag{1b}$$

$$J_-^W=J_+^{W\dagger} \tag{1c}$$

where V^i (A^i) (i=1, 2, --, 15) are vector (axial) currents transforming as an adjoint representation 15 and V^0 is a vector current which is an SU(4) singet, forming a 16-plet together with V^i (i=1, .., 15).

In order to clarify the assumptions needed to derive each of the relations, we discuss them in several steps. First, the relations which follow only from the transformation property of the currents and hadron states under SU(4), are obtained (Sec. II). Secondly, we add the assumptions that the only important t-channel effects are those with non-exotic quantum numbers, and study their consequences (Sec. III). These two steps of assumptions, being general enough, lead to many relations not only among the deep inelastic structure functions but also among various production cross sections including those of charmed hadron* productions.

The next step is to assume an explicit SU(4) symmetry for the virtual Compton amplitude and to impose the duality constraint that the exotic contributions in the

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* We use the notation of the charmed particles in Gaillard et. al. (Ref. 4).

s-channel are entirely due to an SU(4) singlet effect in the t-channel (Sec. IV). The absorptive part of the virtual Compton amplitudes (structure functions) is assumed to have two duality components each of which satisfies the positivity condition. This scheme leads to more stringent relations for the nucleon functions than before, which prove to be almost equivalent to those found in a particular type of quark-parton model, i. e., the model in which the parton distribution functions are composed of the valence and sea (SU(4)-singlet) parts. In two Appendices, we list the solution for the non-diffractive component (Appendix A), and the results of the quark-parton model with SU(4) symmetry (Appendix B).

The main outcome of the paper is as follows:

i) SU(4) symmetry imposes stronger constraints on
$F_1^{\gamma n}/F_1^{\gamma p}$, $(F_1^{\gamma n}+F_1^{\gamma p})/(F_1^{\nu n}+F_1^{\nu p})$, and $F_1^{\nu p}/(4F_1^{\gamma n}-F_1^{\gamma p})$
than SU(3);

ii) Although some of the relations found here have already been obtained in particular models,[7] our approach makes it clear exactly what assumptions are needed for deriving each of them;

iii) Since we do not rely on any dynamical assumptions, it follows that the validity of the relations found here, and therefore of many of the results in the quark-parton model, is independent of the validity of particular dynamical properties such as the Bjorken scaling. This point is important in view of a possible breakdown of the Bjorken scaling suggested by a recent lepton-nucleon experiments.[8]

II. Relations Following from the Transformation Properties of the Currents and Hadrons

Let us consider a transformation,

$$U=e^{i\pi(I2+K2)} \tag{2}$$

where I_2 and K_2 are the isospin and the K-spin operators: they are equal to SU(4) generators,[5] F^2 and F^{14}, respectively. In the quark language the operation U corresponds to the simultaneous interchange, $(p \leftrightarrow n)$ and $(\lambda \leftrightarrow p')$. The charged currents of Eqs. (1) transform under U as

$$UJ_{\pm}^{W}U^{-1}=-J_{\mp}^{W}. \tag{3}$$

On the other hand, a hadron state H transforms under U into another state $\tilde{H}$ which we call the mirror state of H with respect to U. Examples of such mirror pair states are tabulated in TableI.

An immediate consequence of Eq. (3) is the following relation for the weak structure functions F_i's,

$$F_i^{\nu H}=F_i^{\bar{\nu}\tilde{H}},\ (i=1,\ 2,\ 3) \tag{4}$$

where H and $\tilde{H}$ are mirror states with respect to U (Table I). In particular, the proton and neutron-structure functions satisfy Eq. (4), without an approximation, $\theta=0$, in contradistinction with the case of the usual Cabibbo current. The charmed currents, Eqs. (1), act on the nucleons as if they were exactly charge symmetric

Table I

Examples of mirror states $(H, \tilde{H})$ with respect to U. Any pair can be read either as $(H, \tilde{H})$ or as $(\tilde{H}, H)$. The phase accompanying the transformation U is not taken into account in this table.

(p, n),	(Σ^+, C_1^0),	(Σ^0, C_1^+),	(Σ^-, C_1^{++}),
(Ξ^0, X_d^+),	(Ξ^-, X_u^{++}),	(Λ, C_0^+),	(X_s^+, T^0),
(π^+, π^-),	(π^0, π^0),	(D^+, K^-),	$(D^0, \bar{K}^0)$,
(F^+, F^-),	(D^-, K^+),	$(\bar{D}^0, K^0)$,	
$(\psi(3, 1), \phi)$,	(ω, ω),	(ρ^+, ρ^-),	(ρ^0, ρ^0)

though they are actually not. The charm bearing piece of the charged currents of Eqs. (1) acts to recover the relation Eq. (4), not to violate it, in contradiction to the arguments based only on the isospin.[9]

Eqs. (3) lead also to a number of relations among the cross sections where the final hadronic system is completely (exclusive cross sections) or partially (inclusive cross sections) specified. They are conveniently summarized in the formula,

$$\sigma_T(\bar{\nu}H \rightarrow \mu^+ H_f X) = \sigma_T(\nu\tilde{H} \rightarrow \mu^- \tilde{H}_f X), \tag{5}$$

where σ_T denotes the transverse cross section, H and $\tilde{H}$ targets, H_f and $\tilde{H}_f$ observed final hadrons, and X stands for the vacuum for exclusive processes and "anything else" for inclusive processes. One can construct many relations combining them: for instance, one gets*

$$\sigma_T(\nu N \rightarrow \mu^- C_1^{++} + \ldots) = \sigma_T(\bar{\nu} N \rightarrow \mu^+ \Sigma^- + \ldots) \tag{6}$$

and similar relations using other pairs of Table I.

From the transformation property of each piece in the weak currents Eqs. (1) under U, it also follows that, for an isoscalar nucleon target N, the $(\Delta Y = 1)$ production by neutrino is equal to the $(\Delta C = 0)$ production by antineutrino; the $(\Delta Y = 0)$ production by neutrino is equal to the $(\Delta C = -1)$ production by antineutrino; the $\begin{pmatrix}\Delta Y = 0 \\ \Delta C = 1\end{pmatrix}$ production by neutrino is equal to the $\begin{pmatrix}\Delta Y = -1 \\ \Delta C = 0\end{pmatrix}$ production by antineutrino.

III. Assumptions of Non-Exotic T-Channel Exchanges

The purpose of this section is to study the consequences of adding the assumption that the only important t-channel effects are those with "non-exotic" quantum numbers. In the t-channel of the virtual Compton scattering with SU(4) currents, we define those quantum numbers corresponding to the singlet 1 or the adjoint representation 15 to be nonexotic, and all others exotic. Moreover the coupling of the external currents to t-channel exchanges is assumed to obey the 16-plet scheme, which is analogous to the nonet coupling in SU(3) case.** Another assumption which

* N in Eq. (6) denotes any target composed of an equal number of protons and neutrons.

** Since the current Eq. (1a) has an SU(4) singlet piece, the t-channel of the (electromagnetic) compton amplitude contains a 15 coming from $1\otimes15$ as well as two 15's (15_D and 15_F) deriving from $15\otimes15$. We assume that the 15 coming from $1\otimes15$ (or $15\otimes1$) is identical to 15_D (symmetric 15) coming from $15\otimes15$; similarly, the singlet arising from $1\otimes1$ is identified with the singlet from $15\otimes15$. This is achieved in Eq. (7), allowing the suffices a, b and c to run from 0 to 15 with the definition, $d^{0\alpha\beta} = (1/\sqrt{2})\,\delta_{\alpha\beta}$

will be used throughout the paper is that the vector-vector and axial-axial contributions are equal in the deep inelastic limit. Because of this, the expressions for neutrino processes in the following contain a factor 2 relative to those for photon-induced processes.

Under these assumptions, the structure functions $F_1(\omega, q^2)$ are written as

$$(F_1)^{ab}_{\alpha\beta} = i f^{abc} F^c_{\alpha\beta} + d^{abc} D^c_{\alpha\beta} \tag{7}$$

where a and b specify the currents and α and β hadrons; f^{abc} and d^{abc} are the SU(4) structure constants.[10] The electromagnetic and weak structure functions are then given as*

$$(F_1{}^{\gamma})_{\alpha\beta} = \frac{10\sqrt{2}}{9} D^0{}_{\alpha\beta} + 2/3 D^3{}_{\alpha\beta} - 2/3 D^A{}_{\alpha\beta}$$

$$(F_1{}^{\nu})_{\alpha\beta} = 4(\sqrt{2} D^0{}_{\alpha\beta} - F^3{}_{\alpha\beta} + F^A{}_{\alpha\beta}) \tag{8}$$

$$\text{with } F^A_{\alpha\beta} \equiv -\frac{1}{\sqrt{3}} F^8_{\alpha\beta} + \sqrt{\frac{2}{3}} F^{15}_{\alpha\beta}$$

and similarly $D^A_{\alpha\beta}$ in terms of $D^8_{\alpha\beta}$ and $D^{15}_{\alpha\beta}$. Notice that the weak structure functions are independent of the Cabibbo angle θ.

Under the transformation, Eq. (2), the third and A-components** of a 15 simply change the sign, while a singlet remain unchanged. Therefore, taking $\alpha=\beta=H$ and $\tilde{H}$ (mirror pair with respect to U) in Eq. (8) and Eq. (9), one finds immediatey an equality (i=1, 2),

$$F_i{}^{\gamma H} + F_i{}^{\gamma \tilde{H}} = \frac{5}{18}(F_i{}^{\nu H} + F_i{}^{\nu \tilde{H}}). \tag{9}$$

This relation for the proton and neutron appears to be well satisfied by the experimental data,[11] and the prediction is that Eq. (9) should continue to hold even at high energies where all the charm thresholds will be open. Notice that the corresponding relation in SU(3) is an inequality.

One finds also

$$4 \geqq F_i{}^{\gamma H}/F_i{}^{\gamma \tilde{H}} \geqq 1/4 \quad (i=1,\ 2) \tag{10}$$

from Eq. (8), using the positivity of $D^c{}_{\alpha\beta}$'s. This inequality is formally the same as the familiar inequality,[1),3)] and indeed identical to it for the protonn and neutron.

The relations Eq. (9) and inequality (10) have been obtained in the quark-parton model and in the light-cone algebra (see Appendix B).[7] However, they follow in any theories which make the assumptions used above, and of more general validity than the quark-parton model itself (or the light-cone algebra).

It is easy to generalize the above results to inclusive cross section where the observed final hadron is in the target fragmentation region. In such a situation we have, e. g.

* The structure function for incident antineutrino is obtained simply by changing the sign of $F^3{}_{\alpha\beta}$ and $F^A{}_{\alpha\beta}$ in Eq. (8).

** We define the A-component of a 15 to be
$\lambda^A = -(1/\sqrt{3})\lambda^8 + \sqrt{2/3}\,\lambda^{15}$

$$\sigma_T(\gamma H \to H_f X) + \sigma_T(\gamma \tilde{H} \to \tilde{H}_f X)$$
$$= \frac{5}{18}\{\sigma_T(\nu H \to \mu^- H_f X) + \sigma_T(\nu \tilde{H} \to \mu^- \tilde{H}_f X)\}, \tag{11}$$

which is Eq. (9) applied to a complex target $(H\tilde{H}_f)$.

IV. SU(4) Symmetry and Duality Constraints

In this section we add duality constraints to the assumptions made in (II) and (III). We restrict ourselves to nucleon targets, which are assigned to one of 20-dimensional representation of SU(4).* There are two components in the virtual Compton amplitude: one —"diffractive component"— contains only singlets (of SU(4)) in the t-channel, and the other —"non-diffractive component"— contains 1 and 15 in the t-channel and 4*, 20 and 20' in the s-channel. The positivity is imposed on each of the components, which is a dynamical assumption contained in our definition of duality. The results will be compared to those in the quark-parton model, which are derived in Appendix B.

We begin by expanding the virtual Compton amplitudes from the proton and neutron into the eigenamplitudes of SU(4) in the s- (or t-) channel, using the known C-G coe ffcients.[12),13)] The result is given in Table II.

Next, we find the duality solution for the non-diffractive component of the amplitudes. This is done by writing the s-t crossing equations,[13)] setting all the exotic amplitudes (20", 45*, 45 and 84 in the t-channel and 36*, 60* and 140 in the s-channel) to be zero, and solving the equations for the remaining amplitudes. The solution is given in Appendix A, together with the positivity condition for the independent amplitudes. It leads to the following relations (i=1, 2)

$$1/4 \leqq F_i^{\gamma n}/F_i^{\gamma p} \leqq 3/2 \tag{12a}$$
$$F_i^{\nu p}/(4F_i^{\gamma n} - F_i^{\gamma p}) = 6/5 \tag{12b}$$
$$F_i^{\gamma p} - F_i^{\gamma n} = (1/6)(F_i^{\bar{\nu} p} - F_i^{\bar{\nu} n}) \tag{12c}$$
$$= (1/6)(F_i^{\nu n} - F_i^{\nu p})$$

besides those obtained already in (II) and (III).

The upper bound of (12a) is stronger than the corresponding result in SU(3) case (which is $13/(5+2\sqrt{3})$ (Ref. 2). Eq. (12c) is the local form of a well known relation[14)] and is unaffected when one goes from SU(3)[2)] to SU(4). The equality Eq. (12b), as well as Eq. (9) of (III), is peculiar to SU(4) symmetry and stronger than the corresponding SU(3) relations. One can use Eq. (12b), Eq. (9) and Eq. (4) to determine the four kinds of weak structure functions in terms of electromagnetic structure functions as (i=1, 2)

$$F_i^{\nu p} = F_i^{\bar{\nu} n} = (6/5)(4F_i^{\gamma n} - F_i^{\gamma p})$$
$$F_i^{\nu n} = F_i^{\bar{\nu} p} = (6/5)(4F_i^{\gamma p} - F_i^{\gamma n}) \tag{13}$$

Eqs. (12) and Eq. (13) can also be obtained in the quark-parton model of valence-sea type[7)], as shown in Appendix B, and are not a peculiarity of our approach. However, since no dynamical assumptions are made in the latter, any eventual breaking of the

* For the notation see Gaillard et. al. (Ref. (4)); V. Rabl et. al. (Ref. (12)).

Table II

Coefficients in the expansion of the virtual Compton amplitudes into eigenamplitudes of SU(4) in the s-channel. θ is the Cabibbo angle appearing in Eq. (1). $20_{0,0}$ refers to $1+20\to 1+20$; $20_{1,0}$ and $20_{2,0}$ refer to $15+20\to 1+20$ ($20_{0,1}$ and $20_{0,2}$ to $1+20\to 15+20$); all others to $15+20\to 15+20$.

process \ eigen-amplitudes	140	60*	36*	20'	$20_{2,2}$	$20_{1,1}$	$20_{1,2}$ ($20_{2,1}$)	$20_{0,0}$	$20_{0,2}$ ($20_{2,0}$)	$20_{0,1}$ ($20_{1,0}$)	4*
γp	$\frac{61}{72}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{2}{9}$	$\frac{2}{39}$	$\frac{121}{936}$	$\frac{11}{78\sqrt{3}}$	$\frac{2}{9}$	$-\frac{2}{3\sqrt{39}}$	$-\frac{11}{18\sqrt{13}}$	0
γn	$\frac{61}{72}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{2}{9}$	$\frac{2}{39}$	$\frac{121}{936}$	$\frac{11}{78\sqrt{3}}$	$\frac{2}{9}$	$\frac{2}{3\sqrt{39}}$	$\frac{11}{18\sqrt{13}}$	0
νp $(=\bar{\nu} n)$	$\frac{25}{6}\cos^2\theta + \frac{161}{36}\sin^2\theta$	$\cos^2\theta + \frac{3}{4}\sin^2\theta$	$\frac{3}{2}\cos^2\theta + \frac{3}{4}\sin^2\theta$	$\frac{4}{3}\cos^2\theta + \frac{4}{9}\sin^2\theta$	$\frac{16}{39}\cos^2\theta$	$\frac{289}{468}\sin^2\theta$	$-\frac{34\sqrt{3}}{117}\sin^2\theta$	0	0	0	0
νn $(=\bar{\nu} p)$	$\frac{47}{18}\cos^2\theta + \frac{26}{9}\sin^2\theta$	$2\cos^2\theta + \frac{5}{2}\sin^2\theta$	$\frac{3}{2}\cos^2\theta + \frac{3}{5}\sin^2\theta$	$\frac{4}{9}\cos^2\theta + \frac{2}{9}\sin^2\theta$	$\frac{16}{39}\cos^2\theta + \frac{32}{39}\sin^2\theta$	$\frac{121}{117}\cos^2\theta + \frac{133}{234}\sin^2\theta$	$\frac{44}{39\sqrt{3}}\cos^2\theta + \frac{10}{39\sqrt{3}}\sin^2\theta$	0	0	0	$\frac{2}{5}\sin^2\theta$

Bjorken scaling in deep inelastic limit should occur satisfying Eq. (13). Furthermore, the relations containing different types of structure functions F_1 and F_3 (such as those in quark-parton model: see Eq. (B. 3)) cannot be obtained in our approach. Finally, we add a comment on our assumption about the coupling of the singlet component of electromagnetic current. We have simply set equal (see the footnote of p. 204) the two symmetric 15 amplitudes in the t-channel, one for $15+20\rightarrow15+20$ and the other for $1+20\rightarrow15+20$. Alternatively, one could require the Iizuka-Okubo-Zweig rule relations for the non-diffractive part of structure functions, i. e., assume that there is no non-zero matrix elements between nucleon states which contain $\phi(\lambda\bar{\lambda})$-or $\psi(c\bar{c})$- component as an external line. This leads again to Eq. (A. 4) and consequently to the same results, (12). Without an assumption which fixes the coupling of singlet component relative to 15-plet component in the electromagnetic current, the two-component duality and positivity only would not lead to any of the relations, (12).

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Appendix A. Solution for the Non-Diffractive Component

The solution for the non-diffractive part of the Compton amplitudes, discussed in (IV), is given by (for the ampitude, $15+20\rightarrow15+20$)

$$A_s(4^*)=\frac{5}{3}d_1+\frac{55}{8}d_2$$

$$A_s(20_{2,1})=A_s(20_{1,2})=\frac{44}{13\sqrt{3}}d_1+\frac{177}{52\sqrt{3}}d_2$$

$$A_s(20_{1,1})=-\frac{16}{13}d_1+\frac{56}{13}d_2 \qquad \text{(A. 1)}$$

$$A_s(20')=-3d_1+\frac{23}{8}d_2$$

$$A_t(1)=-\frac{15}{2\sqrt{3}}d_1+\frac{105}{4\sqrt{3}}d_2$$

$$A_t(15_{1,F})=-\frac{2}{3}\sqrt{\frac{61}{2}}d_1 \qquad \text{(A. 2)}$$

$$A_t(15_{2,F})=-2\sqrt{\frac{61}{6}}d_2$$

with $d_1\equiv\sqrt{\frac{6}{61}}A_t\ (15_{1,D})$

$$d_2\equiv\sqrt{\frac{2}{61}}A_t\ (15_{2,D}) \qquad \text{(A. 3)}$$

where $A_s(R_{rr'})\equiv\langle{}^{out}R_r|M_s|R_{r'},\ {}^{in}\rangle$, *etc.*

Moreover, the assumption about the coupling of current-current vertex (see the footnote of p. 204) leads to ($A_s(20_{0,0})$ refers to $1+20\rightarrow1+20$; $A_s(20_{1,0})$ and $A_s(20_{2,0})$

to 15+20→1+20),

$$A_s(20_{0,0})=-\frac{d_1}{4}+\frac{7}{8}d_2$$

$$A_s(20_{1,0})=-\frac{4}{\sqrt{13}}d_1-\frac{5\sqrt{13}}{52}d_2 \qquad \text{(A. 4)}$$

$$A_s(20_{2,0})=\frac{5\sqrt{39}}{156}d_1-\frac{12}{\sqrt{39}}d_2.$$

The positivity condition, imposed on the independent amplitudes d_1 and d_2, is given by

$$-33/8\leq d_1/d_2\leq 23/24,$$

$$d_2\geq 0. \qquad \text{(A. 5)}$$

The easiest way to see Eq. (12b) and Eq. (12c) is to observe that the relation among A_t (15)'s given in Eq (A. 2) is equivalent* to

$$D^3_{\alpha\beta}=F^3_{\alpha\beta} \text{ and } D^A_{\alpha\beta}=F^A_{\alpha\beta} \qquad \text{(A. 6)}$$

($\alpha=\beta=$proton) in Eq. (8). The inequality (12a) can be obtained by using Table II, Eq. (A. 1), Eq. (A. 4) and (A. 5).

Appendix B. Results of the Quark-Parton Model With SU(4) Symmetry

In this Appendix we derive the relations among the structure functions which follow from the quark-parton model. The structure functions are written in terms of the parton distribution functions as usual, with additional charmed pieces (we assume the fourth charmed quark has charge 2/3). Following the procedure of Nachtmann (Ref. 1), we first obtain the positivity conditions on the parton distribution functions: they are given by**

$$N_\lambda=N_c\geq 0$$

$$3N_n-2N_\lambda\geq 0 \qquad \text{(B. 1)}$$

$$10N_p-5N_n-2N_\lambda\geq 0$$

$$N_{\bar c}=N_{\bar\lambda}$$

$$N_{\bar p}\geq 0 \qquad \text{(B. 2)}$$

$$2N_{\bar n}-N_{\bar p}\geq 0$$

$$8N_{\bar\lambda}-4N_{\bar n}-N_{\bar p}\geq 0.$$

Then the following relations among the structure functions can be found in a straightforward manner: (i=1, 2)

$$1/4\leq F_i^{\gamma n}/F_i^{\gamma p}\leq 13/7$$

$$(18/5)F_i^{\gamma N}=F_i^{\nu N}=F_i^{\bar\nu N} \qquad \text{(B. 3)}$$

$$2/5\leq F_i^{\nu p}/(4F_i^{\gamma n}-F_i^{\gamma p})\leq 66/35$$

$$6(F_1^{\gamma p}-F_1^{\gamma n})=F_3^{\nu p}-F_3^{\nu n}.$$

If one restricts oneself to the model in which the quark distribution functions can be separated in two parts: valence part (satisfying itself (B. 1) and (B. 2)) plus

* Notice the difference in the normalization between the structure constants and the corresponding CG coeffcients.

** We follow the notation of Nachtmann (Ref. 1).

SU(4) singlet sea (to which each quark contributes equally), then one finds, instead of (B. 3), (i=1, 2)

$$\begin{aligned} &1/4 \leqq F_i^{\gamma n}/F_i^{\gamma p} \leqq 3/2 \\ &(18/5)F_i^{\gamma N} = F_i^{\nu N} = F_i^{\bar{\nu} N} \\ &F_i^{\nu p}/(4F_i^{\gamma n} - F_i^{\gamma p}) = 6/5 \\ &6(F_1^{\gamma p} - F_1^{\gamma n}) = F_3^{\nu p} - F_3^{\nu n}. \end{aligned} \qquad \text{(B. 4)}$$

Some of the results of (B. 3) and (B. 4) have been obtained already (Ref. 7).

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